Heterogenous Agents in the Sequence Space

Ludwig Straub

AEA, January 2025

The "heterogeneity revolution" in NK macro

- Over the last 10 years, heterogeneous-agent New-Keynesian ("HANK") models have become staple of modern macro

  [Oh-Reis 2012, McKay-Nakamura-Steinsson 2016, Guerrieri-Lorenzoni 2017, Kaplan-Moll-Violante 2018, …]

- **What are they so useful for?**

  1. Definitely useful for analyzing **heterogeneity itself**!
  2. Possibly useful because heterogeneity affects **aggregate outcomes**

- **Today:** Focus on (2), work in the **sequence space !**

How heterogeneity matters for aggregates

1. Heterogeneous households in partial equilibrium
2. A “canonical” HANK model
3. Fiscal policy
4. Monetary policy
5. Other fun things you can do!

My slides build on joint work with Adrien Auclert, Matt Rognlie, and several other great coauthors! For more, see recent Annual Review of Econ article.

Heterogeneity in partial equilibrium

The Ricardian representative agent

- Backbone of every textbook NK model: Ricardian representative agent
- E.g. with fixed $r = \beta^{-1} - 1$ and variable income stream Z_t

$$\max_{\{C_t\}} \sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\sigma}}{1-\sigma} \qquad C_t + A_t = (1+r)A_{t-1} + Z_t \qquad A_t \geq \underline{A}$$

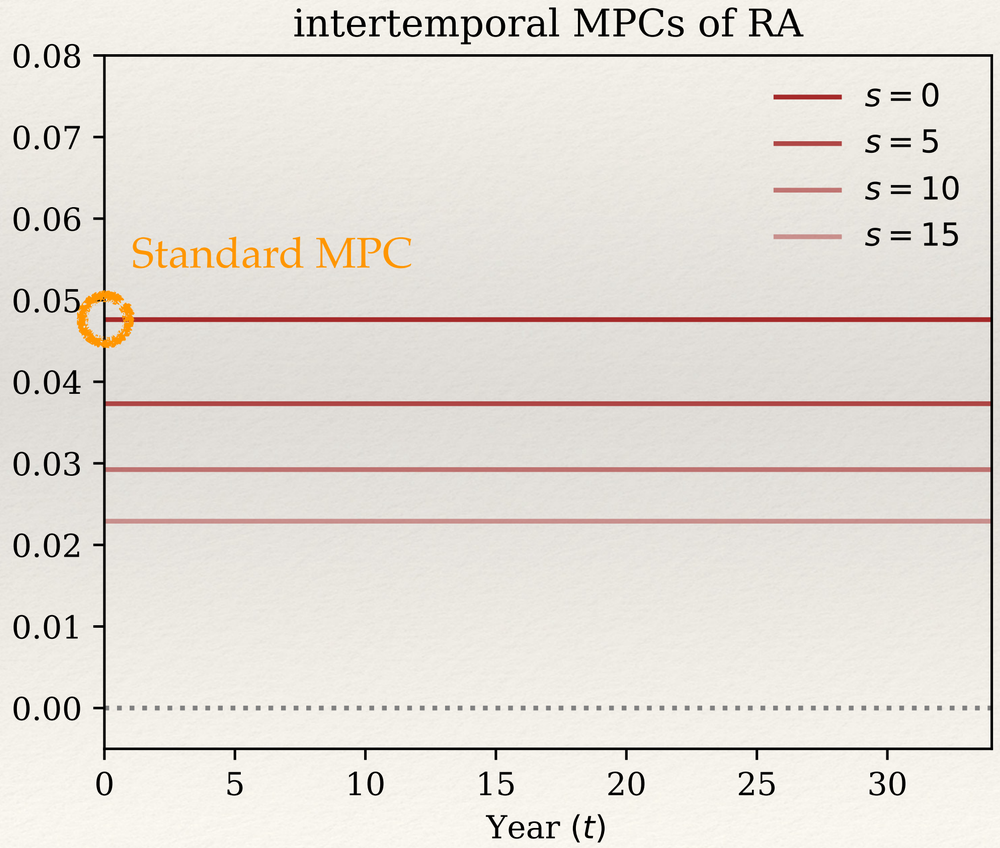
- Given A_{-1}, consumption is function of Z_t …

$$C_t = \mathscr{C}_t\left(\{Z_0, Z_1, Z_2, \ldots\}\right)$$

- What's so special about this function for the representative agent?

Tiny consumption responses to income shocks!

- Responses to income shocks $\partial C_t/\partial Z_s$ for various dates s



Heterogeneous agents

- Now add **idiosyncratic productivity shocks** e_{it}

$$\max_{\{c_{it}\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{c_{it}^{1-\sigma}}{1-\sigma}$$

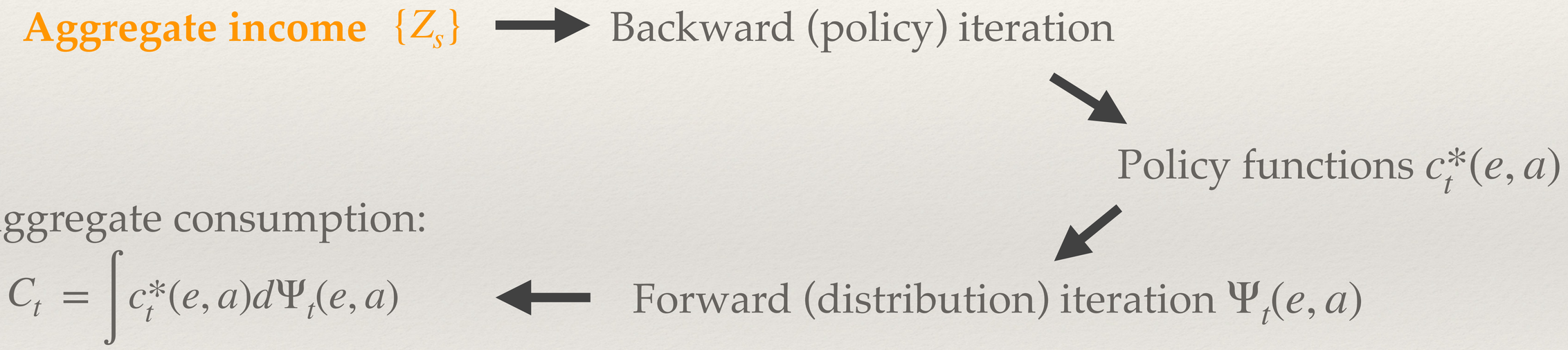
$$c_{it} + a_{it} = (1+r)a_{it-1} + e_{it}Z_t \qquad a_{it} \geq 0 \qquad C_t \equiv \int c_{it} di$$

- Given initial distribution $\{a_{i,-1}\}$, consumption is still *some* function of Z_t …

$$C_t = \mathscr{C}_t\left(\{Z_0, Z_1, Z_2, \ldots\}\right)$$

The intertemporal consumption function

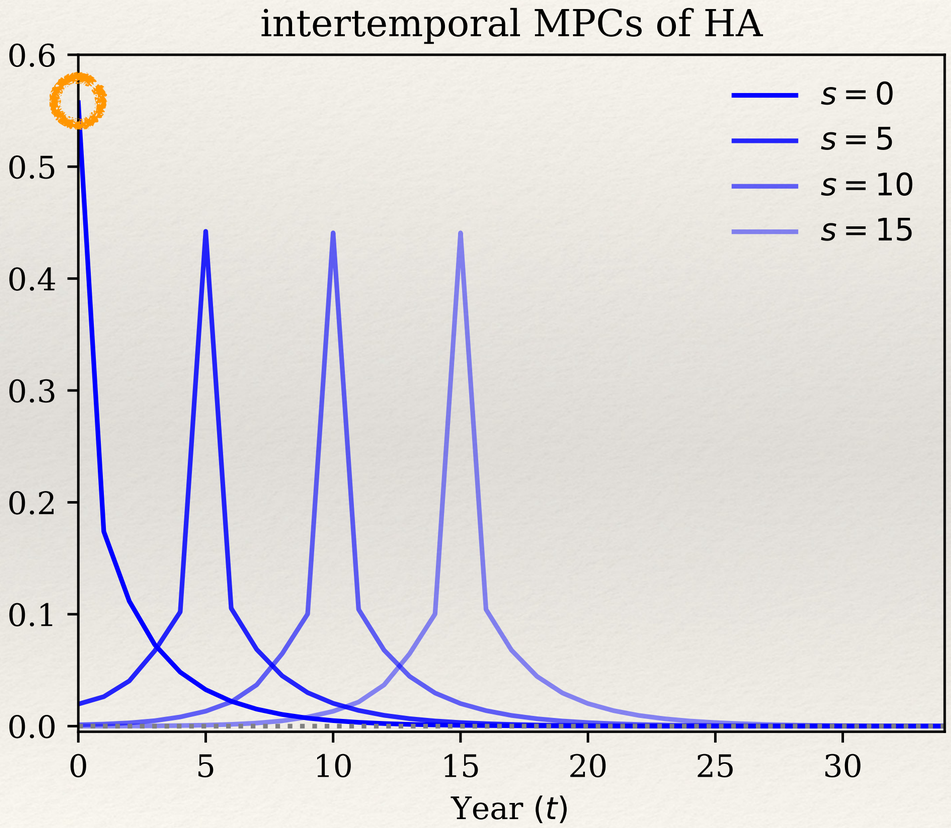
- The HA consumption function is now a little more complicated …

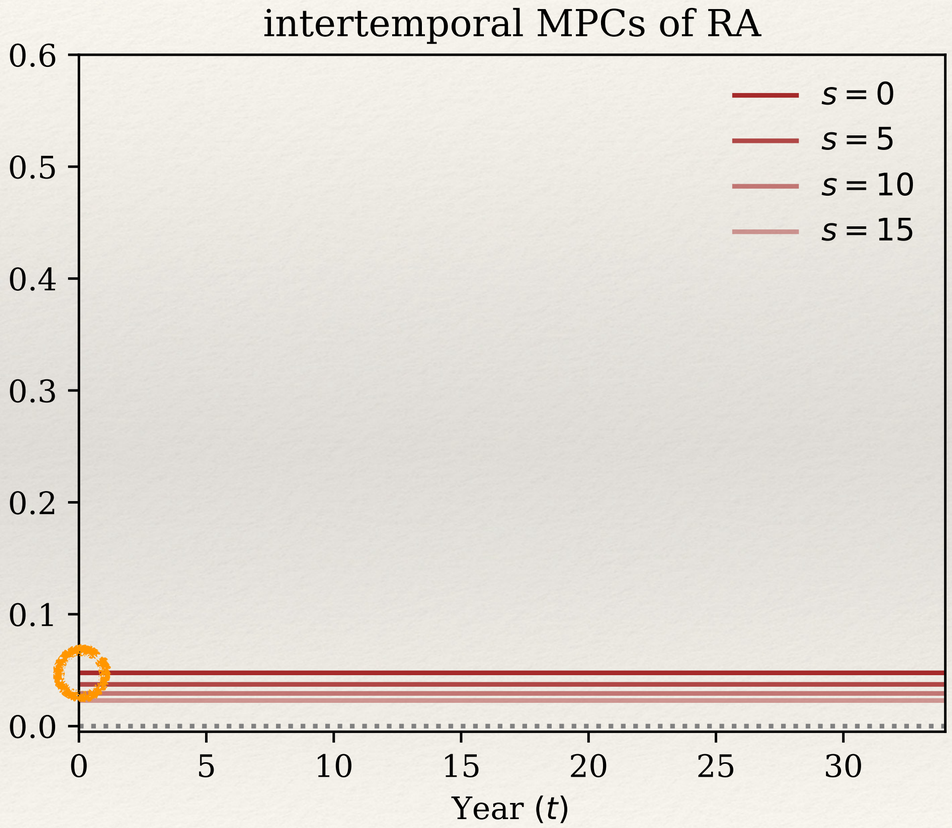


- What if we compute consumption responses to income shocks in this model?

Heterogeneous agents

- Responses to income shocks $\partial C_t / \partial Z_s$ for various dates s

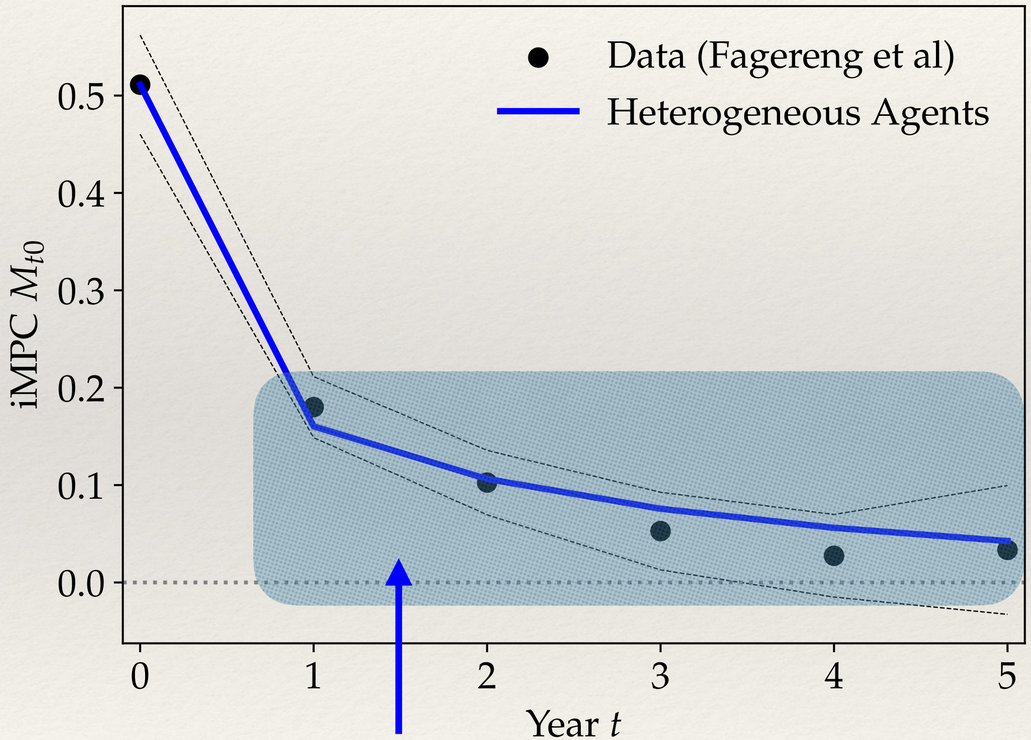




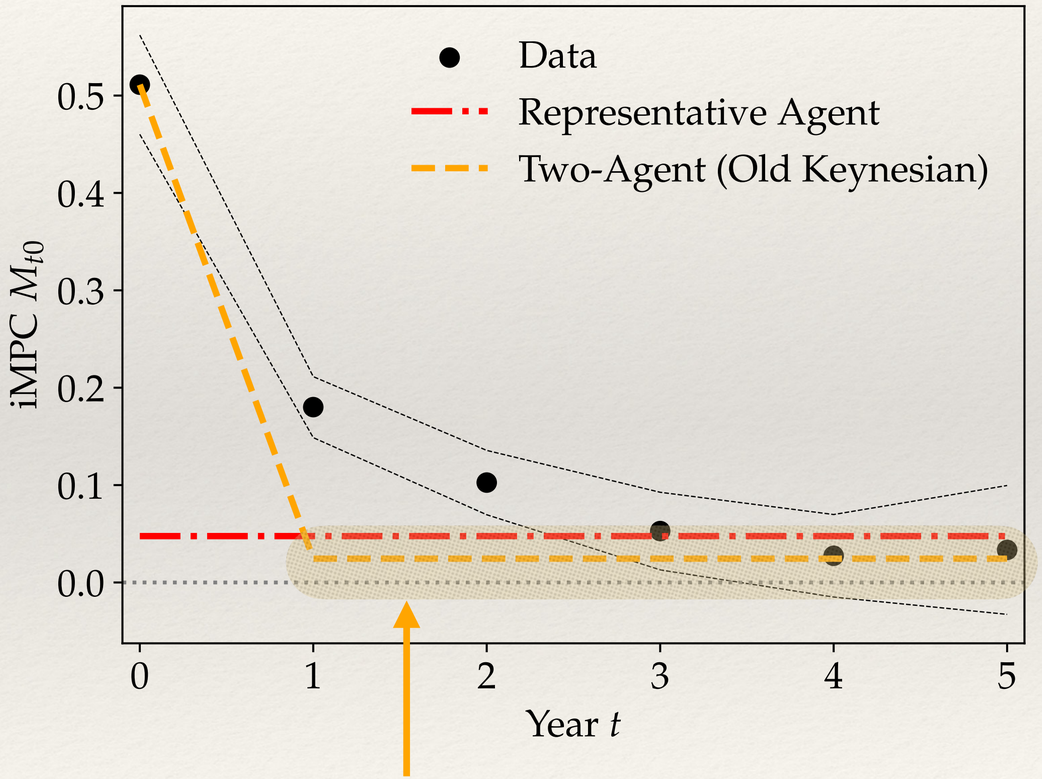
Auclert Rognlie Straub "Intertemporal Keynesian Cross"

Models vs. data

- Can compare $\partial C_t / \partial Z_0$ to data from Fagereng-Holm-Natvik:



HA: Households spend down transfer relatively slowly



TA: Households spend either right away or little

Auclert Rognlie Straub "Intertemporal Keynesian Cross"

Matrix of intertemporal MPCs

- Can stack these responses into a matrix **M**, "intertemporal MPC matrix"
  - formally: matrix derivative ("Jacobian") of the consumption function

$$\mathbf{M} = \begin{pmatrix} \partial C_0/\partial Z_0 & \partial C_0/\partial Z_1 & \partial C_0/\partial Z_2 & \cdots \\ \partial C_1/\partial Z_0 & \partial C_1/\partial Z_1 & \partial C_1/\partial Z_2 & \cdots \\ \partial C_2/\partial Z_0 & \partial C_2/\partial Z_1 & \partial C_2/\partial Z_2 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

Impulse response of **C** to date 0
(unanticipated) increase in income **Z**

Impulse response of **C** to date 1
(anticipated) increase in income **Z**

Matrix of intertemporal MPCs

- Can stack these responses into a matrix $\mathbf{M}$, "intertemporal MPC matrix"
  - formally: matrix derivative ("Jacobian") of the consumption function
- Then for any $d\mathbf{Z} = \left(dZ_0, dZ_1, dZ_2, \ldots\right)'$ consumption response is

$$d\mathbf{C} = \mathbf{M} \cdot d\mathbf{Z}$$

- Like old-Keynesian models where $\Delta C = \text{mpc} \cdot \Delta Z$
- More general theme in HANK later: "resurrects" old-Keynesian features!

A “canonical” HANK model

Making of a canonical HANK model

- The textbook representative-agent NK (RANK) model consists of:
  1. household side: ~~representative agent~~ heterogeneous agents, s.t. income risk
  2. fiscal policy: ~~irrelevant due to Ricardian equivalence~~ No more Ricardian equivalence!
  3. monetary policy: Taylor rule Start with "real interest rate rule"
  4. supply side: linear production, sticky ~~prices set by firms~~ wages set by unions
- Will go over all four ingredients in an economy with perfect foresight
  - without loss to first order (certainty equivalence)

(1) Heterogeneous households

- Exactly like before, except income is now after-tax labor income

$$Z_t = (1 - \tau_t) w_t N_t$$

- Real interest rate r_t possibly time varying
- Intertemporal consumption function now

$$C_t = \mathscr{C}_t\left(\{r_s, Z_s\}\right)$$

(2, 3) Fiscal and monetary policy

- Government sets $\{G_s, T_s\}$ subject to

$$B_t = \left(1 + r_{t-1}\right) B_{t-1} + G_t - T_t \qquad T_t = \tau_t w_t N_t \qquad B_t \text{ bounded}$$

- Central bank sets nominal rate

$$i_t = r_t + \phi \pi_{t+1} \qquad \text{today: } \phi \searrow 1 \qquad \text{i.e. real rate} = r_t$$

(4) Supply side

- Linear production $Y_t = N_t$ with flexible prices, so that real wage $w_t = 1$
- Sticky nominal wages, set by unions

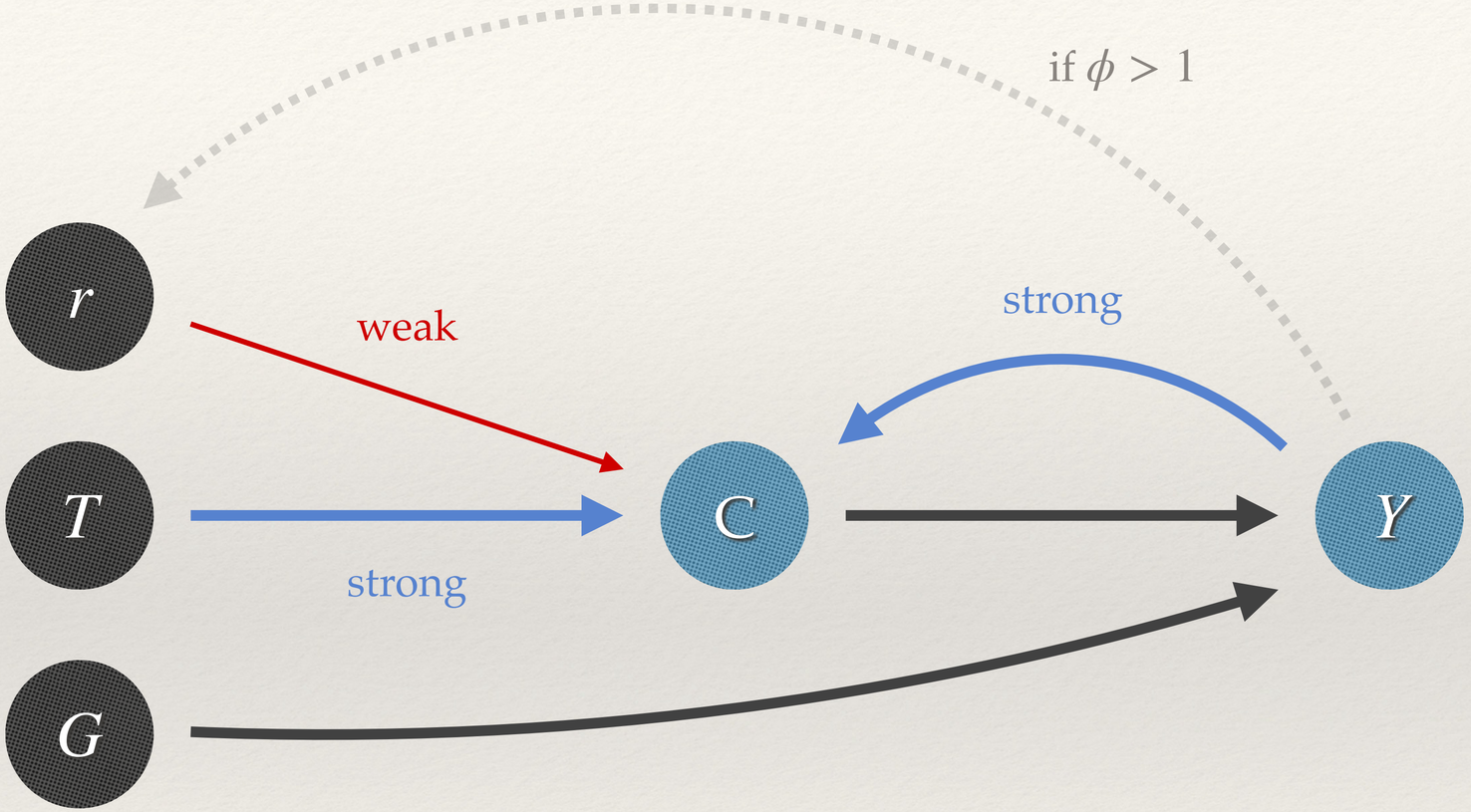
$$\pi_t = \kappa \int \left(v'\left(n_{it}\right) - \frac{\epsilon}{\epsilon - 1}\left(1 - \tau_t\right) w_t e_{it} u'\left(c_{it}\right) \right) di + \beta \pi_{t+1}$$

average gap in households' FOC for labor

  - useful starting point: labor rationed equally $n_{it} = N_t$

- Better than sticky prices + flexible wages (→ countercyclical profits…)
- That's it!

Equilibrium as a graph



$$Y_t = G_t + \mathscr{C}_t\left(\left\{r_s, Y_s - T_s\right\}\right)$$

Fiscal Policy

Intertemporal Keynesian cross

- Imagine central bank keeps real interest rate constant: $r_t = r$.
- Fiscal policy shock $d\mathbf{G} = (dG_0, dG_1, \ldots)$, $d\mathbf{T} = (dT_0, dT_1, \ldots)$, same NPV.
- What happens to **output**, $d\mathbf{Y} = ?$

$$Y_t = G_t + \mathscr{C}_t\left(\{Y_s - T_s\}\right)$$

$$d\mathbf{Y} = d\mathbf{G} + \mathbf{M}d\mathbf{T} + \mathbf{M}d\mathbf{Y}$$

$$dY_t = dG_t + \sum_{s=0}^{\infty} \frac{\partial \mathscr{C}_t}{\partial Z_s}\left(dY_s - dT_s\right)$$

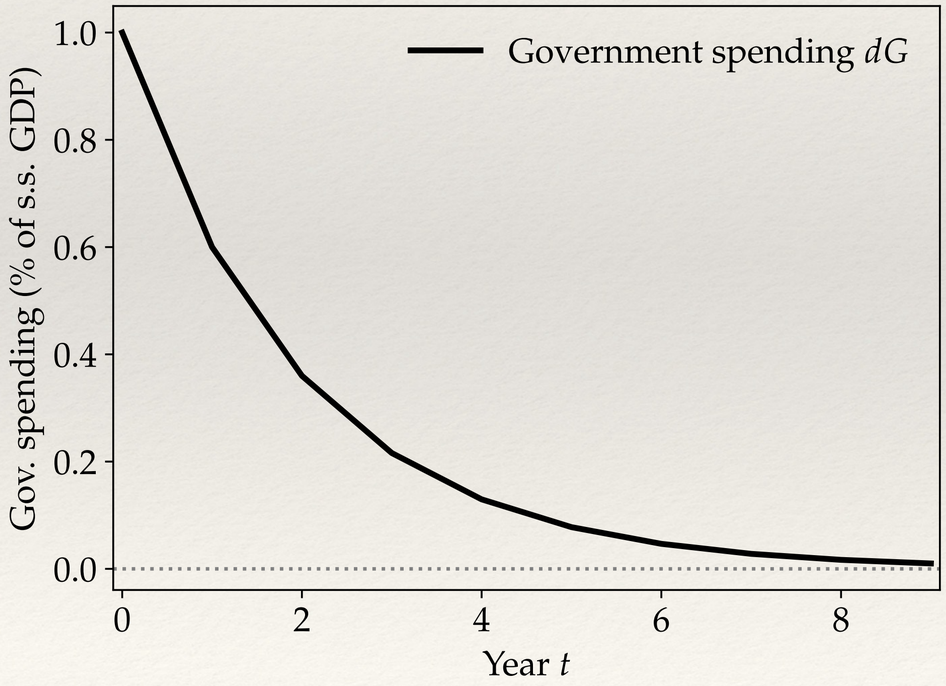
Intertemporal Keynesian cross

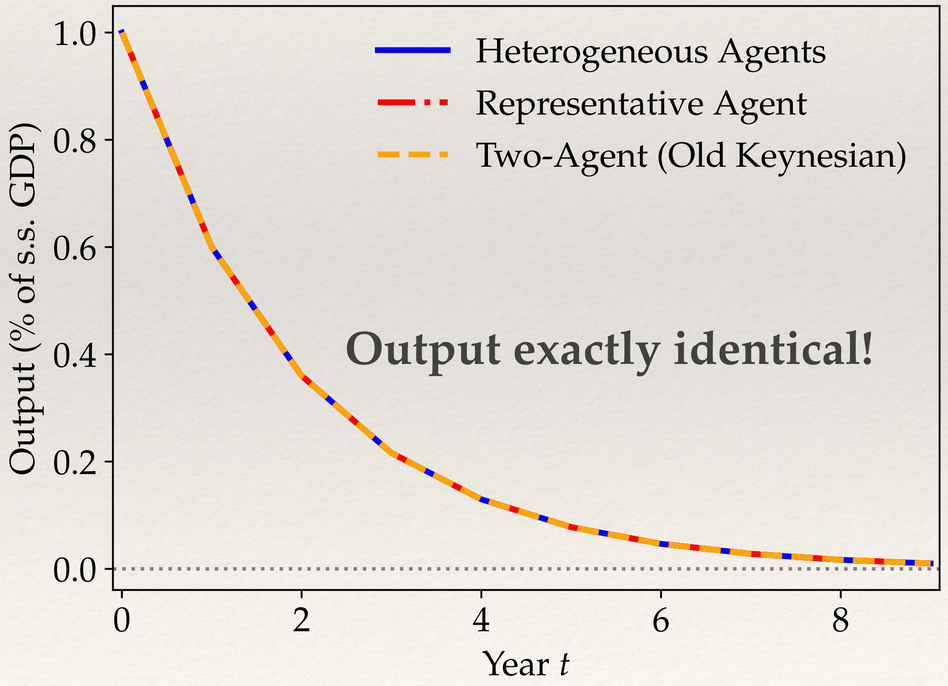
Auclert Rognlie Straub "Intertemporal Keynesian Cross", see also Hagedorn Manovskii Mitman 2019

The Balanced-Budget Multiplier

$$d\mathbf{Y} = d\mathbf{G} - \mathbf{M}d\mathbf{T} + \mathbf{M}d\mathbf{Y}$$

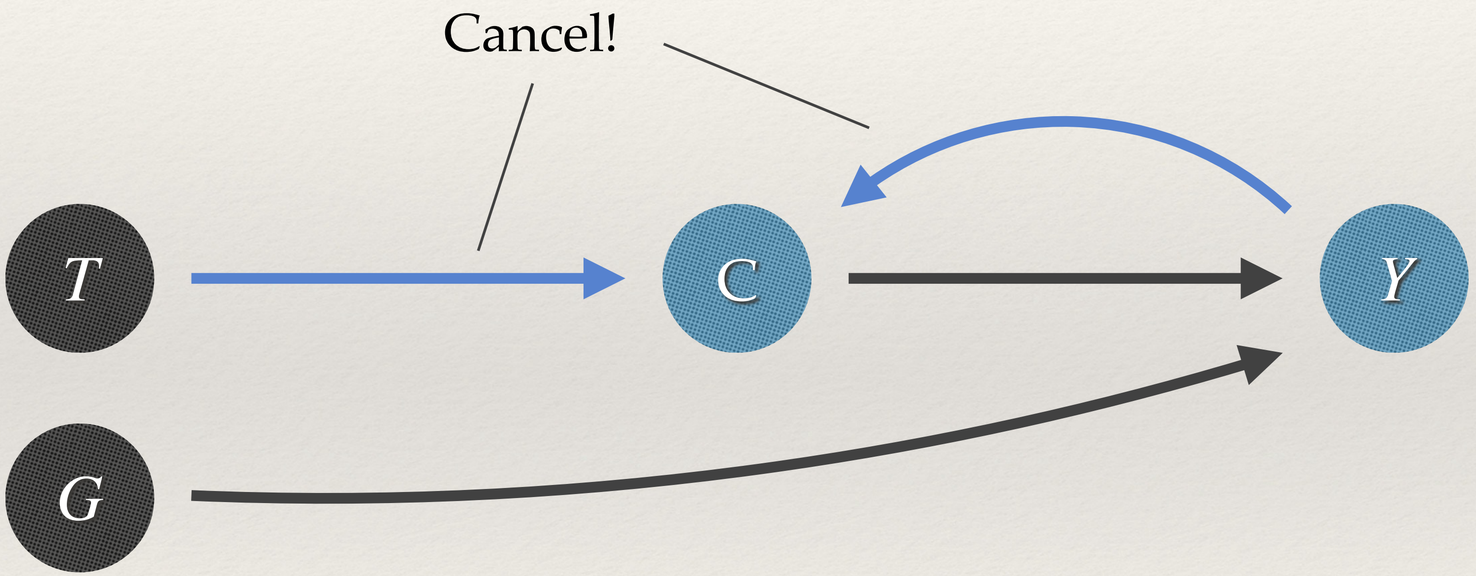
- Imagine the government spends more but raises taxes to pay for it, $d\mathbf{G} = d\mathbf{T}$





cf. Woodford 2011 for RA, Bilbiie 2011 for TA, Auclert-Rognlie-Straub for HA

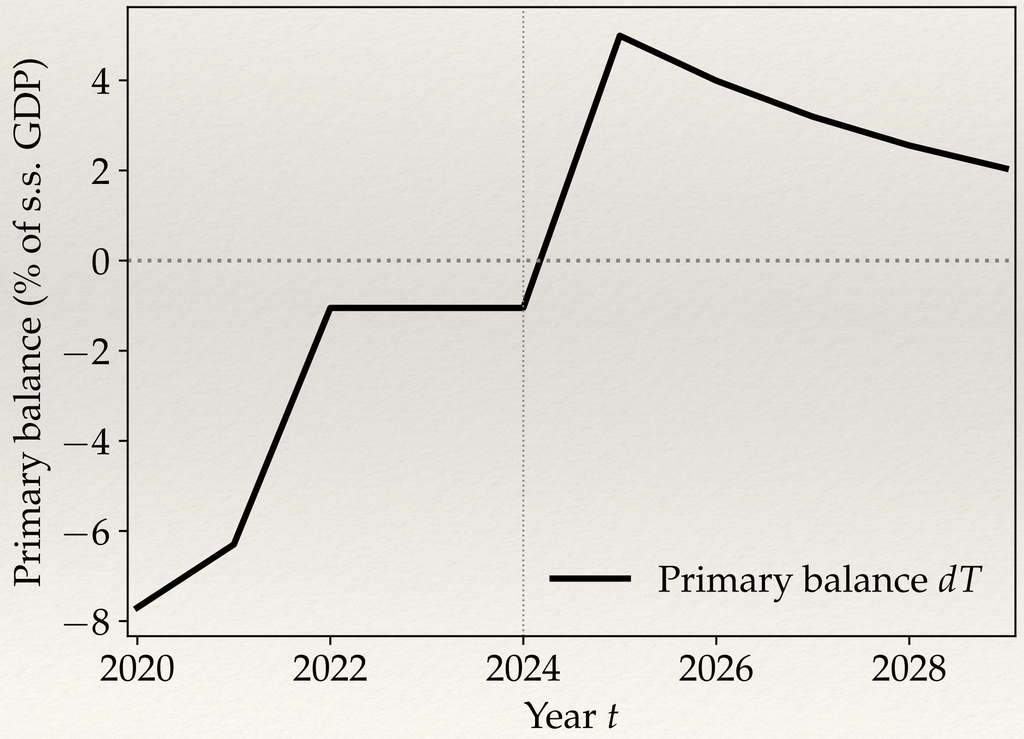
The Balanced-Budget Multiplier



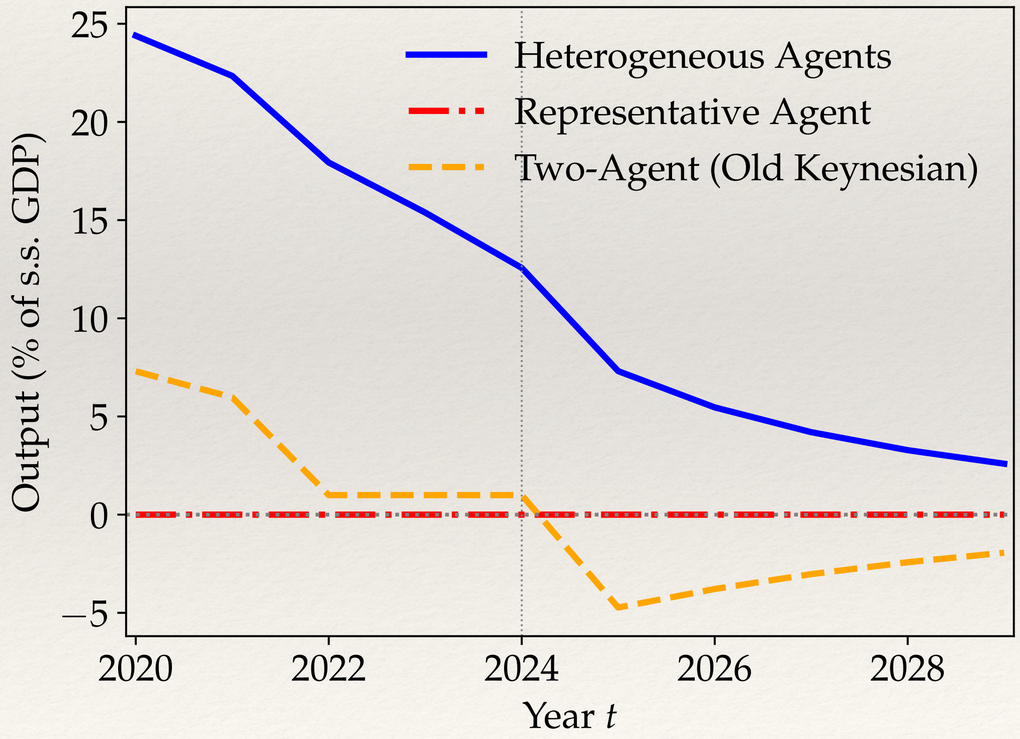
The long shadow of Covid stimulus

- Feed in Covid stimulus and solve for consumption and inflation.

Tax change



Consumption



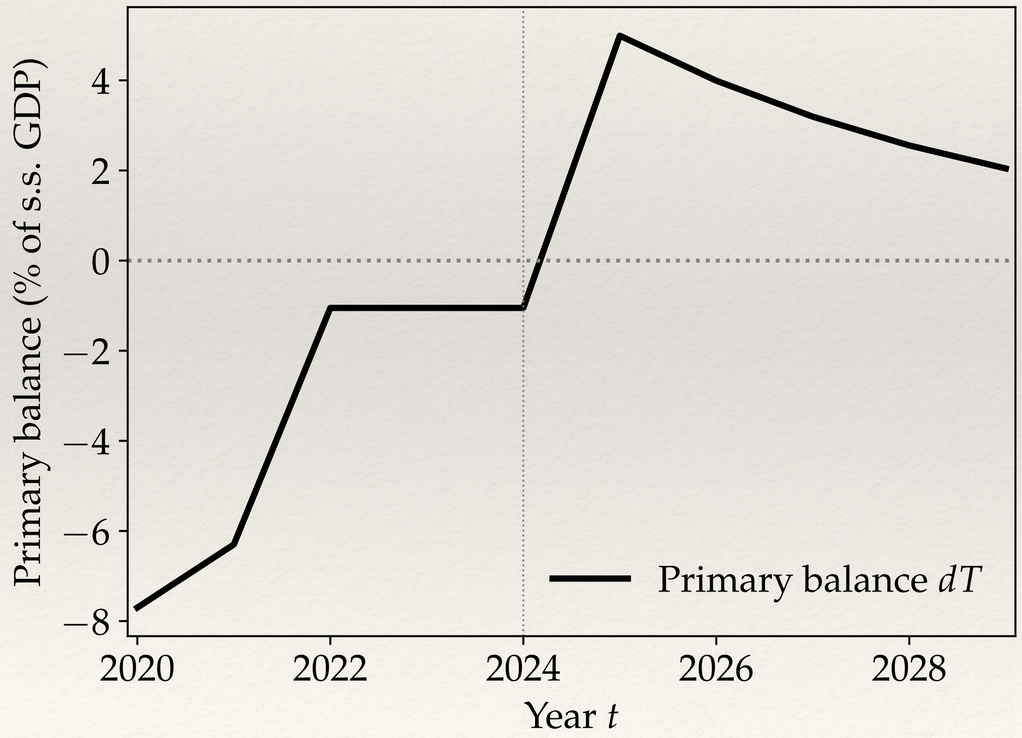
Constructed as 70% of observed primary balance beyond pre-Covid surplus of -2%

See also Bayer et al, Aggarwal et al. 2023, Bardóczy, Sim and Tischbirek 2024

The long shadow of Covid stimulus

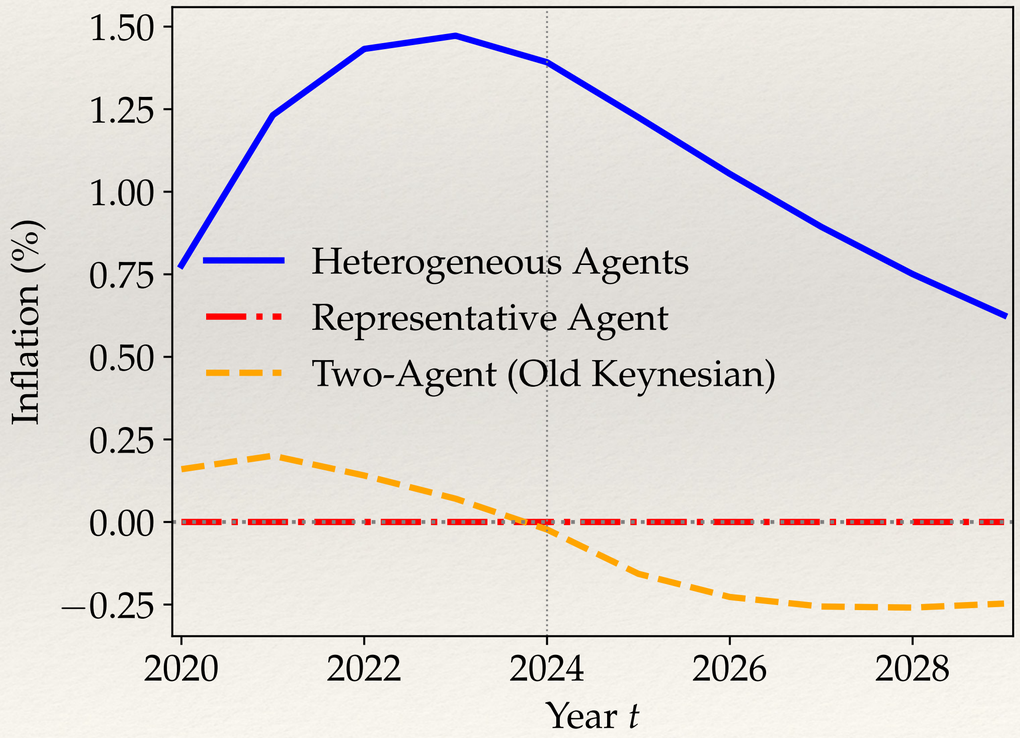
- Feed in Covid stimulus and solve for output and inflation.

Tax change



Constructed as 70% of observed primary balance beyond pre-Covid level -2%

Inflation (relative to target)



Standard hybrid NKPC with 50% weight on lagged inflation, $\kappa = 0.01$

Monetary policy

Monetary policy

- Allow for central bank to shock the real interest rate $\{r_s\}$
- Assume gov. keeps debt repayment $(1 + r_{t-1})B_t$ constant and adjusts T_t

$$Y_t = G_t + \mathscr{C}_t\Big(\{r_s, Y_s - T_s\}\Big) \qquad T_t = (1+r)B + G - \frac{(1+r)B}{1+r_t}$$

Key channels with HA
(Kaplan Moll Violante)

$$d\mathbf{Y} = \mathbf{M}^r \frac{d\mathbf{r}}{1+r} - \mathbf{M}B\frac{d\mathbf{r}}{1+r} + \mathbf{M}d\mathbf{Y}$$

Key channel with RA

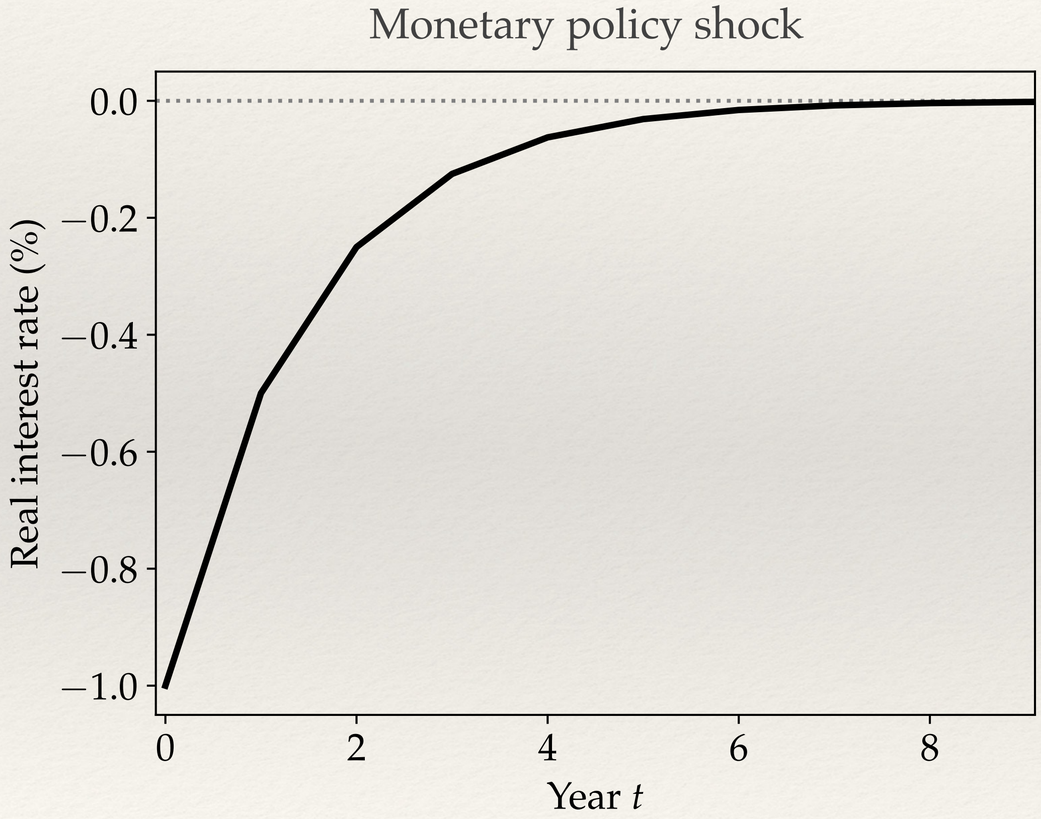
direct effect of r

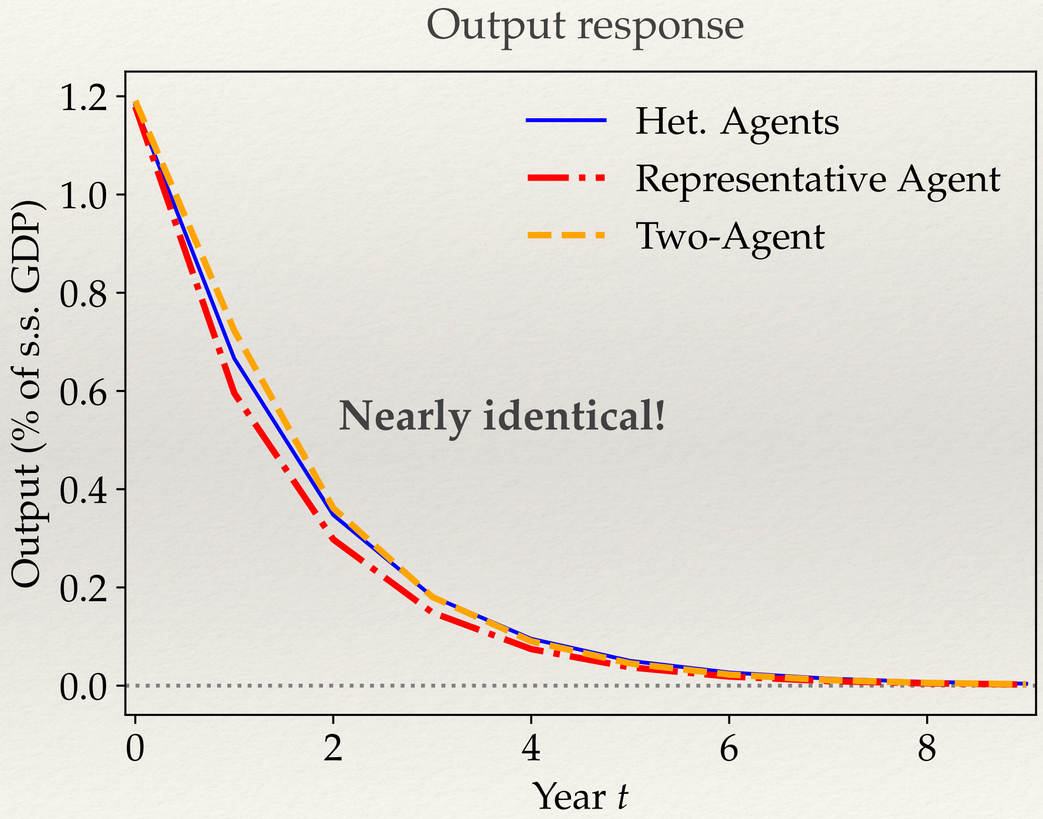
indirect effect via gov. budget

indirect effect via Y

Aggregate effects of monetary policy

- Does HANK matter for aggregate effects of monetary policy?

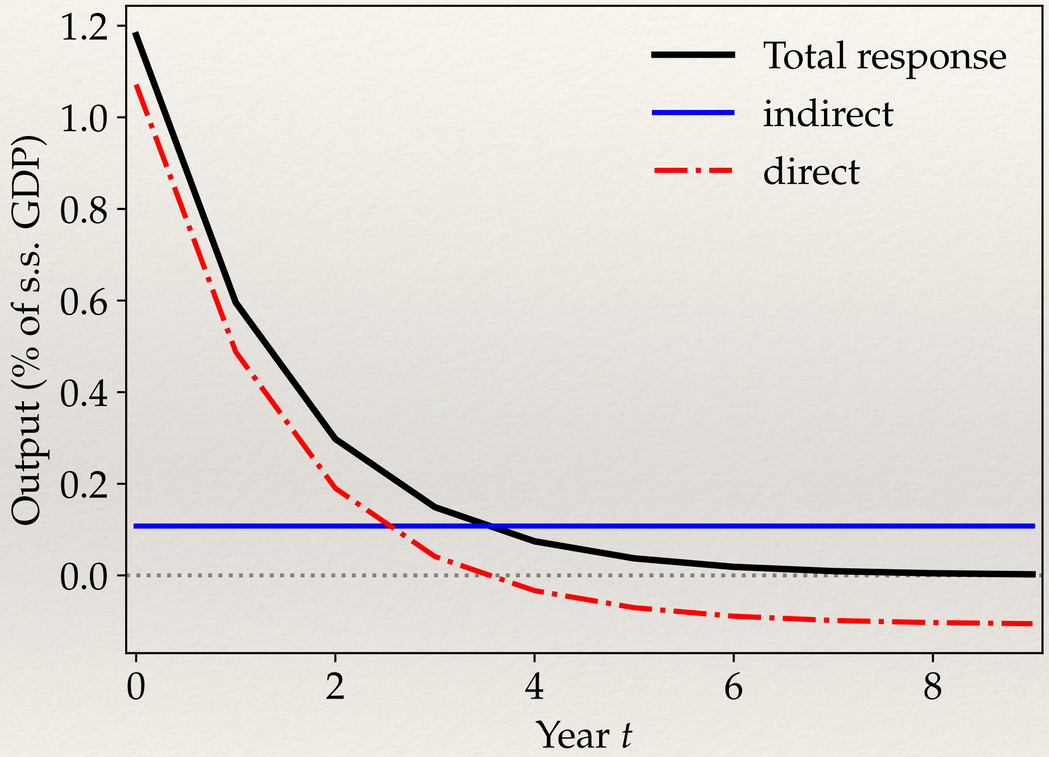




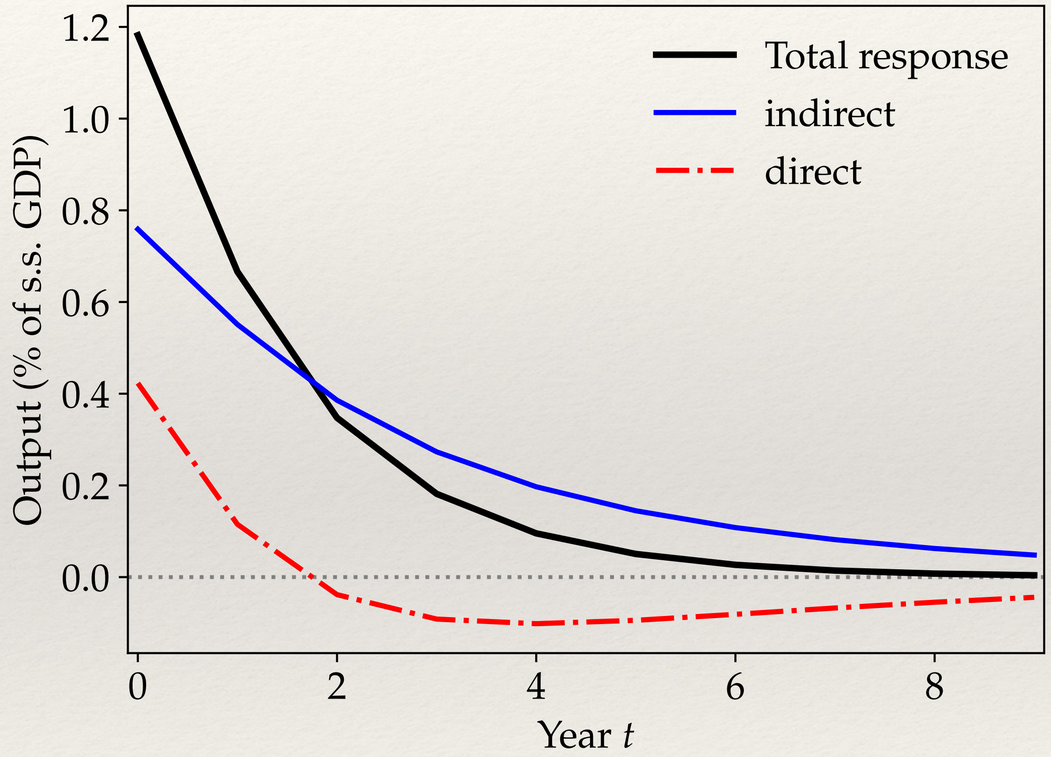
cf. Werning 2015, Auclert Rognlie Straub "Micro Jumps Macro Humps" (Appendix A)

Direct and indirect effects

Representative agent



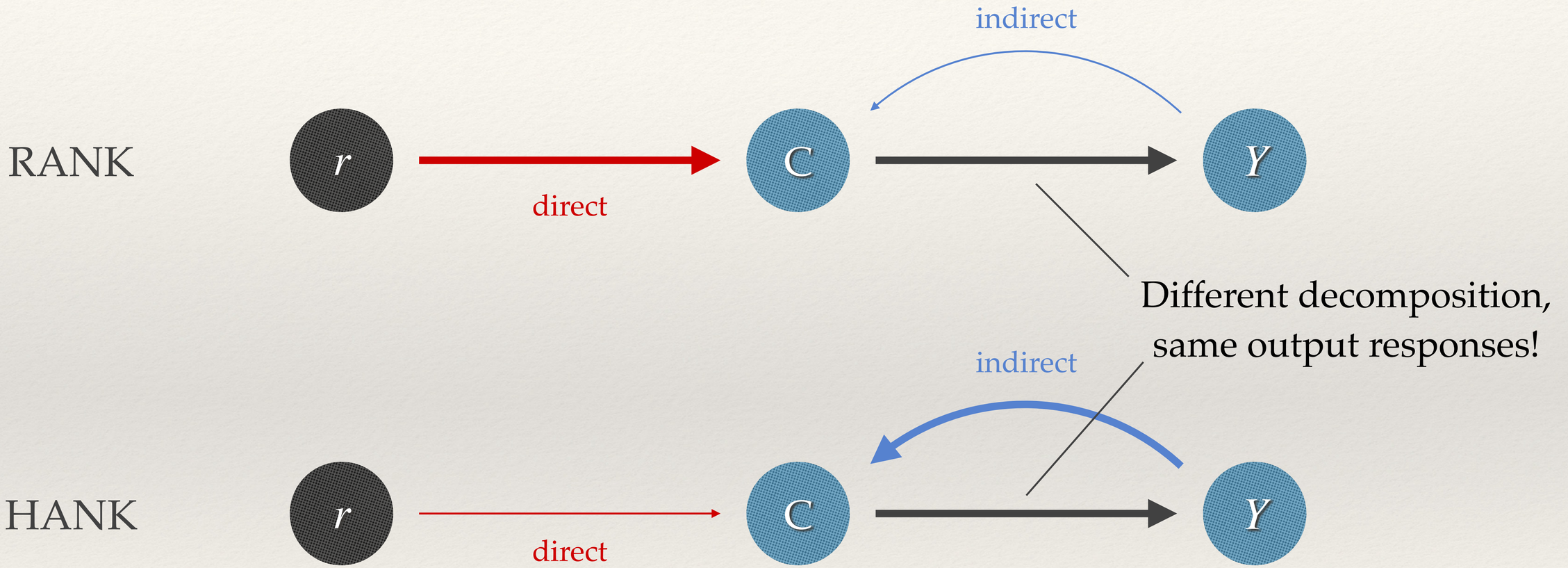
Heterogeneous agents



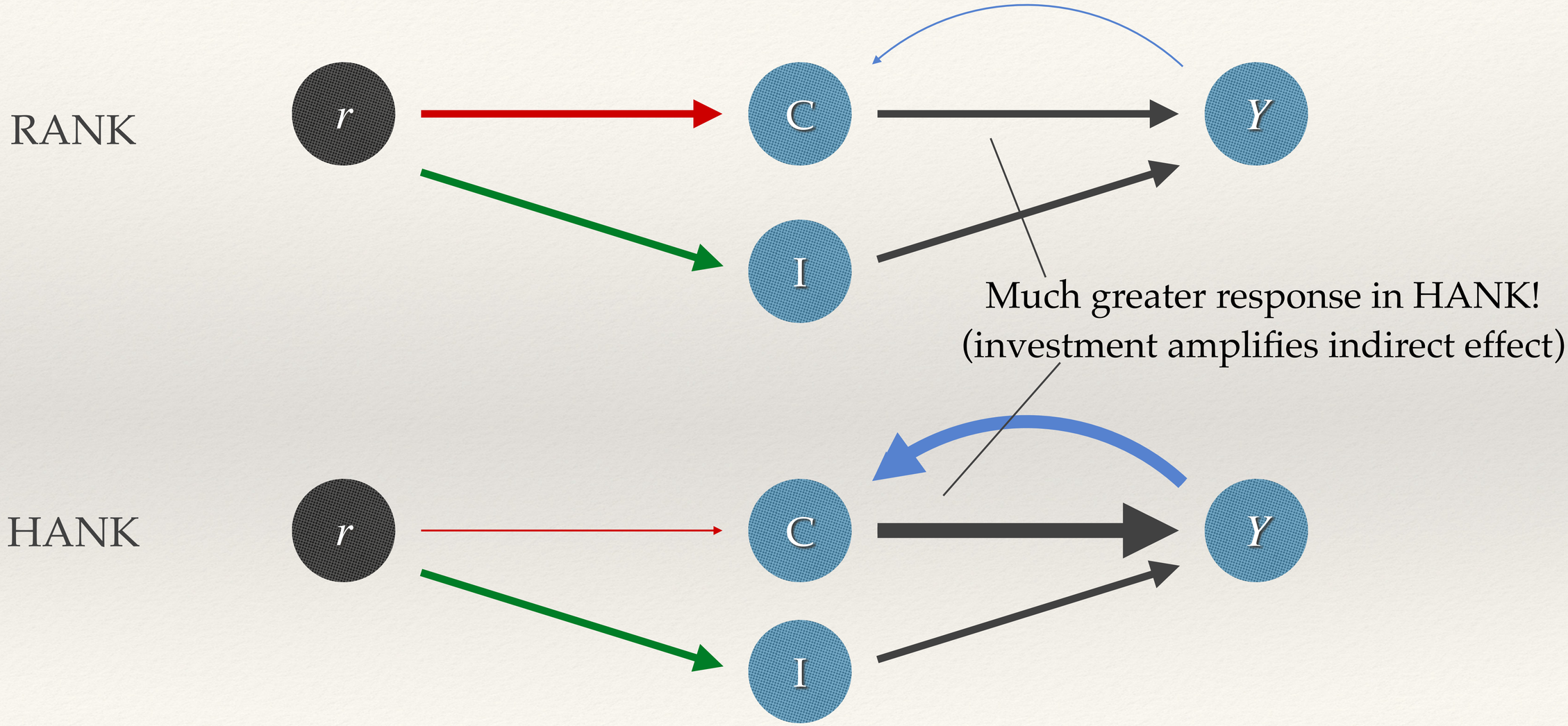
- HANK similar to RANK because stronger indirect offsets weaker direct effect!

[Werning 2015]

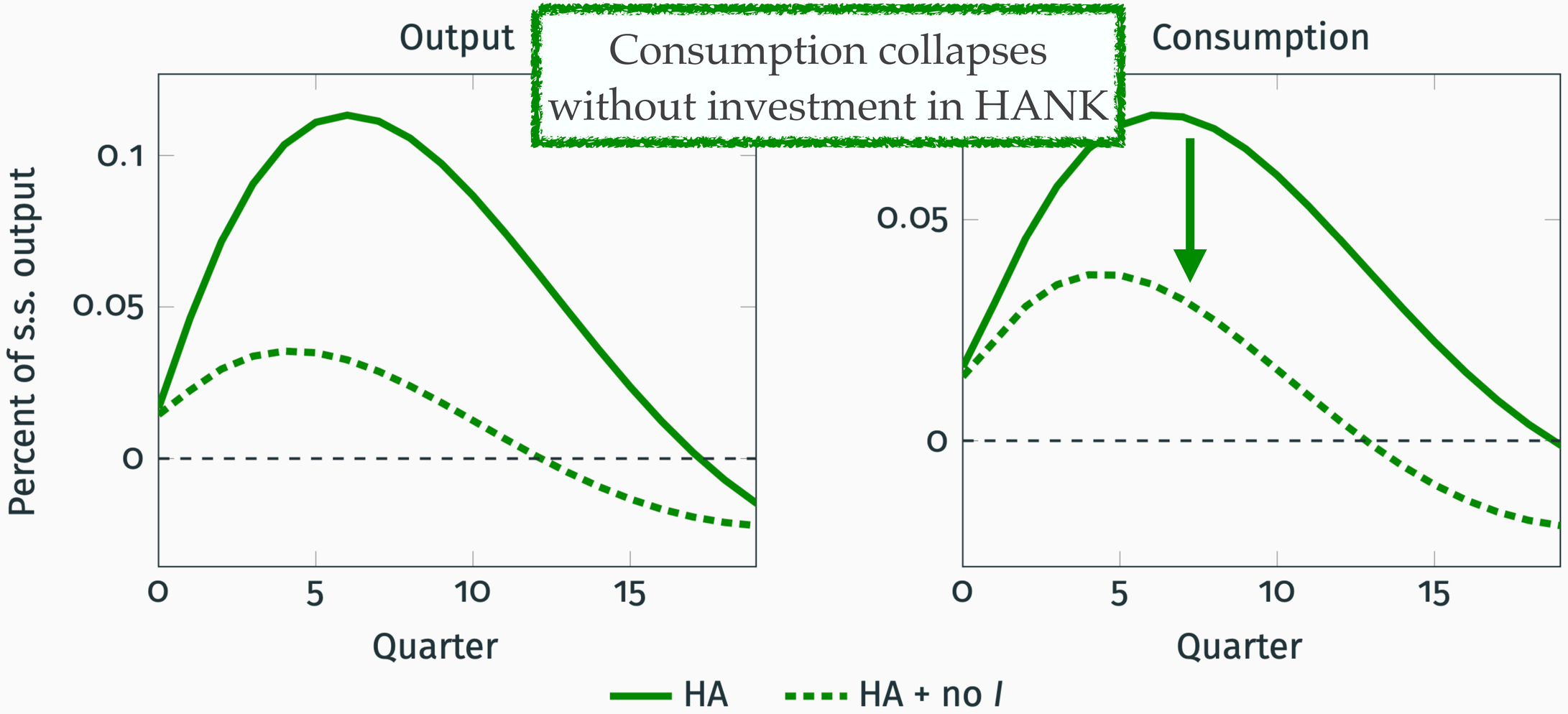
Monetary policy flowcharts



What if there is investment?

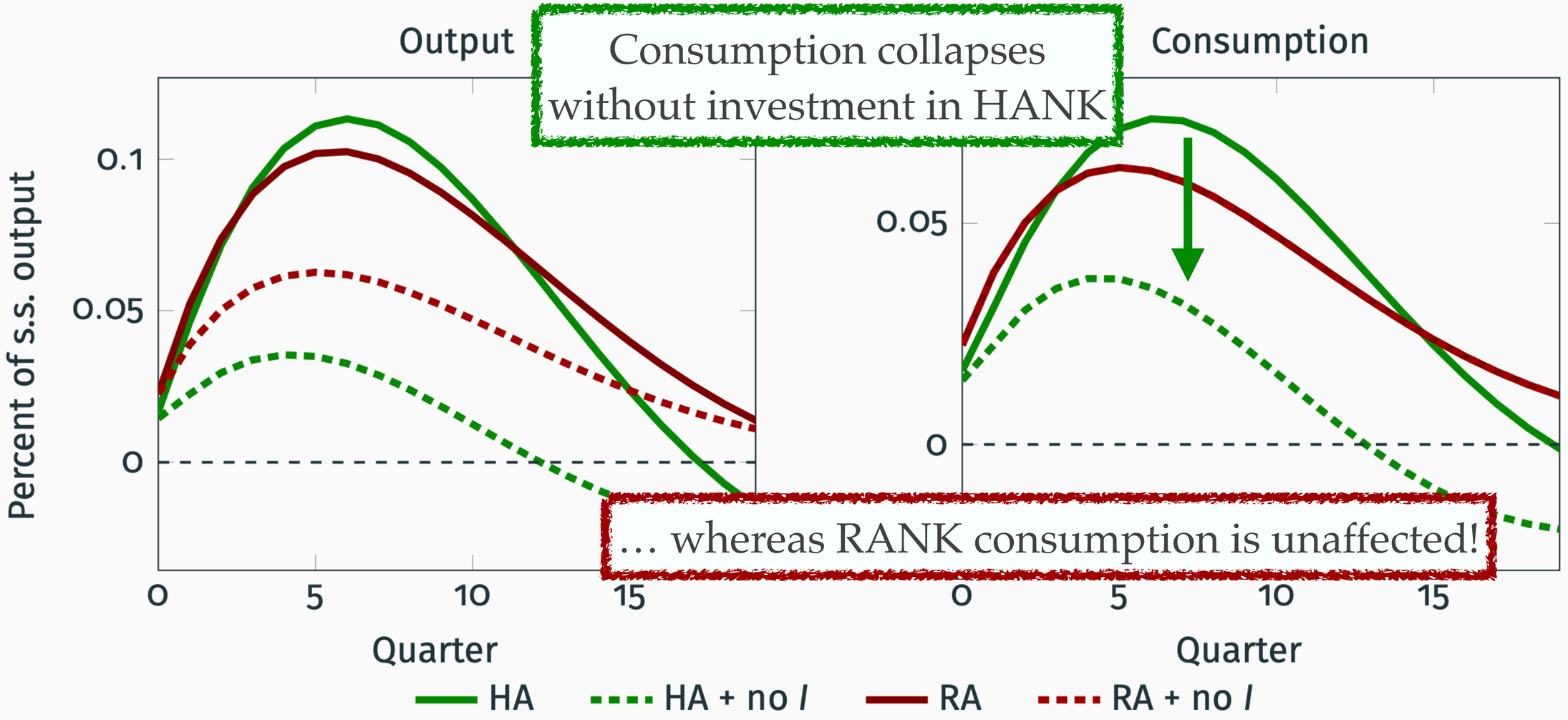


Role of investment in monetary transmission



From Auclert Rognlie Straub "Micro Jumps and Macro Humps"

Role of investment in monetary transmission



From Auclert Rognlie Straub "Micro Jumps and Macro Humps"

Broader implications of investment in HANK

- Limited response to monetary policy tightening
  - if house prices & residential investment doesn't respond much!
- Investment stimulus (e.g. CHIPS, IRA) spills over into consumption!
  - Generates outsized positive effect on aggregate demand

Other fun things

Information frictions

- Lots of interest in **deviations from "FIRE"** (full info rational expectations)

  [e.g. Farhi-Werning 2018, ARS 2020, Pfäuti-Seyrich, Bianchi-Vimercati-Eichenbaum-Guerreiro]
- Integrate in HANK?

- **Example:** What if households are **myopic** and learn of income changes only when they happen? How does it affect fiscal policy?

- Technique generalizes to: sticky info (Mankiw-Reis, Carroll), cognitive discounting (Gabaix), level *k*, noisy information (Angeletos)

Myopic households

- Imagine households learn of income changes only when it happens.
- What is the right **M** ?

$$\mathbf{M} = \begin{pmatrix} M_{00} & M_{01} & M_{02} & \cdots \\ M_{10} & M_{11} & M_{12} & \cdots \\ M_{20} & M_{21} & M_{22} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

Impulse response of **C** to date 0
(unanticipated) increase in income **Z**

Impulse response of **C** to date 1
(anticipated) increase in income **Z**

From Auclert Rognlie Straub "Micro Jumps and Macro Humps"

Myopic households

- Imagine households learn of income changes only when it happens.
- What is the right **M** ?

$$\mathbf{M} = \begin{pmatrix} M_{00} & 0 & M_{02} & \cdots \\ M_{10} & M_{00} & M_{12} & \cdots \\ M_{20} & M_{10} & M_{22} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

Impulse response of **C** to date 0
(unanticipated) increase in income **Z**

Impulse response of **C** to date 1
(anticipated) increase in income **Z**

From Auclert Rognlie Straub "Micro Jumps and Macro Humps"

Myopic households

- Imagine households learn of income changes only when it happens.
- What is the right **M** ?

$$\mathbf{M} = \begin{pmatrix} M_{00} & 0 & 0 & \cdots \\ M_{10} & M_{00} & 0 & \cdots \\ M_{20} & M_{10} & M_{00} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

Impulse response of **C** to date 0
(unanticipated) increase in income **Z**

Impulse response of **C** to date 1
(anticipated) increase in income **Z**

From Auclert Rognlie Straub "Micro Jumps and Macro Humps"

Myopic households

- Imagine households learn of income changes only when it happens.
- What is the right **M** ?

$$\mathbf{M} = \begin{pmatrix} M_{00} & 0 & 0 & \cdots \\ M_{10} & M_{00} & 0 & \cdots \\ M_{20} & M_{10} & M_{00} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix} \equiv \tilde{\mathbf{M}}$$

Myopic **M** matrix

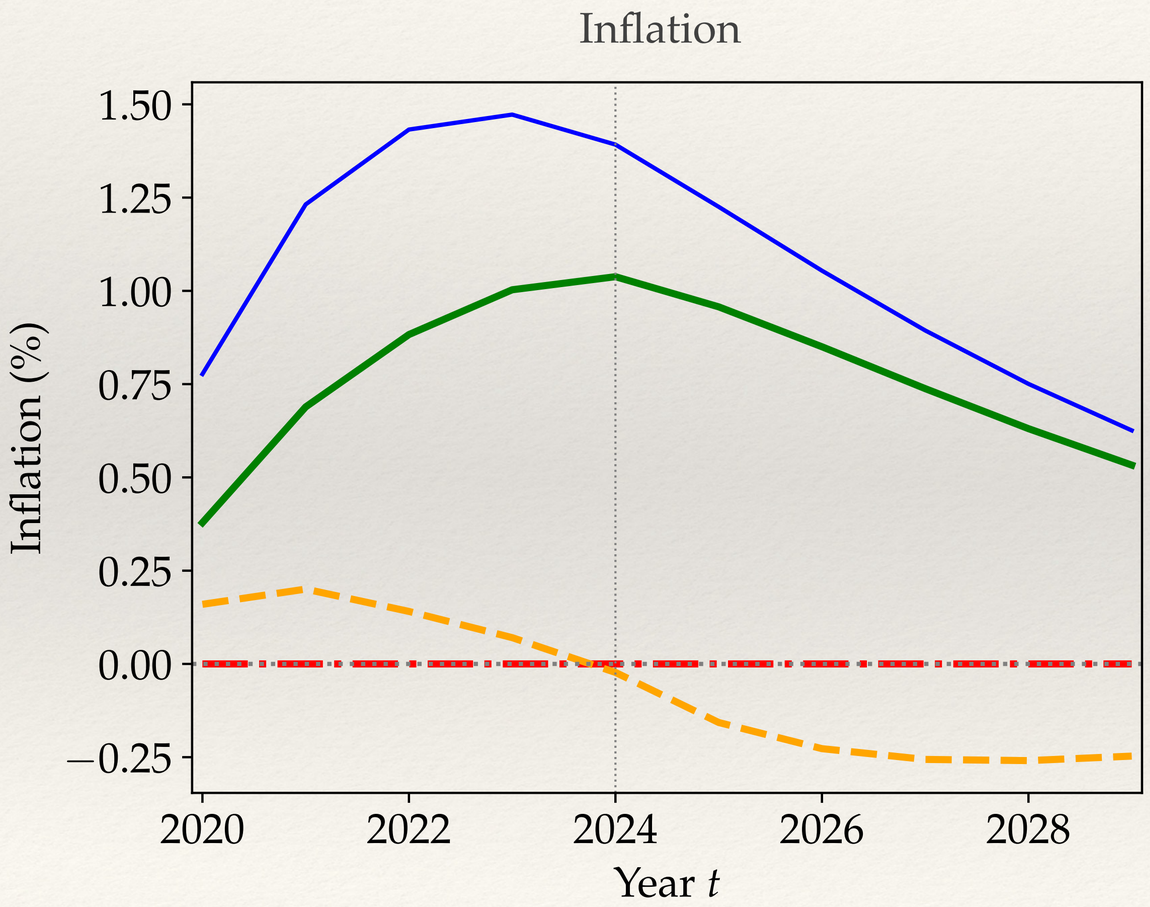
Impulse response of **C** to date 0
(unanticipated) increase in income **Z**

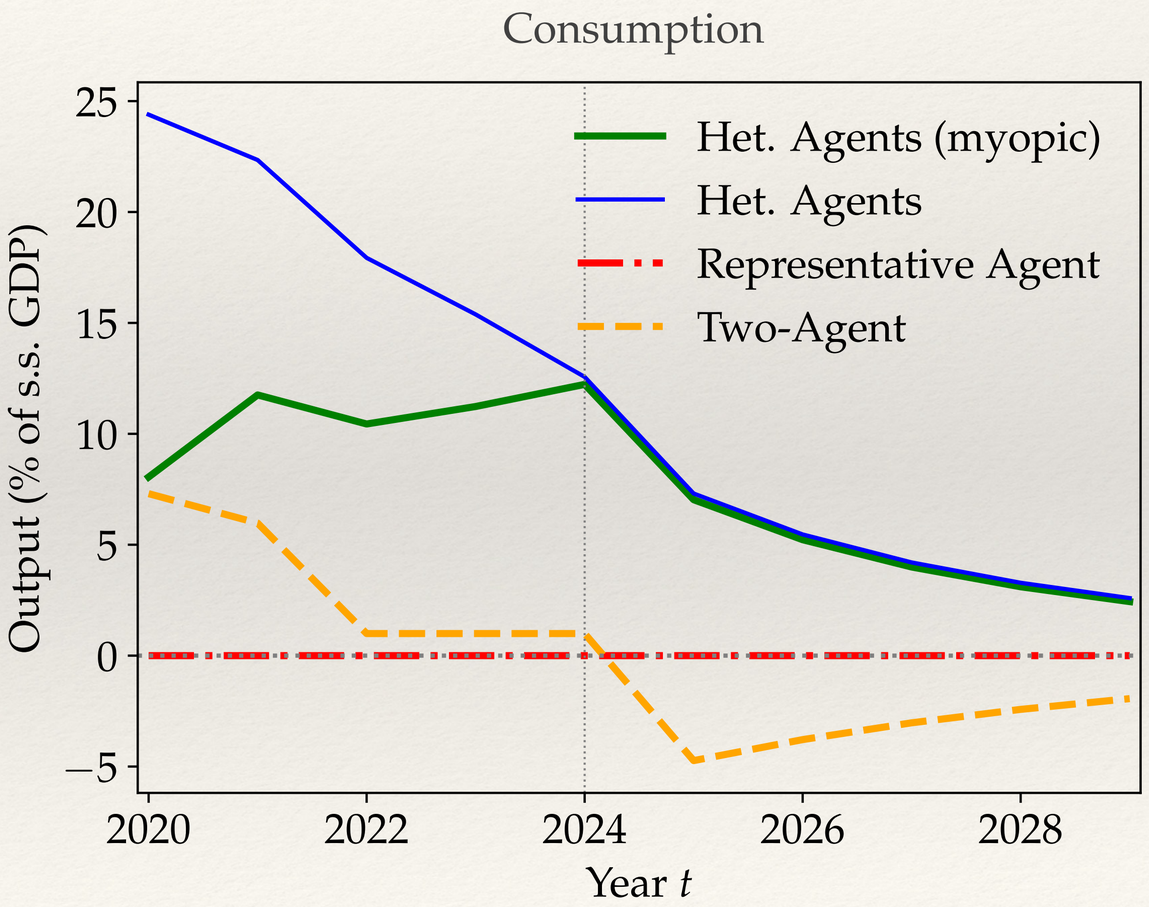
Impulse response of **C** to date 1
(anticipated) increase in income **Z**

From Auclert Rognlie Straub "Micro Jumps and Macro Humps"

Fiscal policy with myopia

$$d\mathbf{Y} = d\mathbf{G} - \tilde{\mathbf{M}} d\mathbf{T} + \tilde{\mathbf{M}} d\mathbf{Y}$$





Semi-structural policy analysis in the sequence space

- Model with both fiscal transfers and monetary policy

$$d\mathbf{Y} = \mathbf{M}^r \frac{d\mathbf{r}}{1+r} - \mathbf{M}B\frac{d\mathbf{r}}{1+r} - \mathbf{M}d\mathbf{T} + \mathbf{M}d\mathbf{Y}$$

- General solution of the form

$$d\mathbf{Y} = \mathbf{G}^T d\mathbf{T} + \mathbf{G}^r d\mathbf{r}$$

- Imagine I get $d\mathbf{Y}^0 = \mathbf{G}^T d\mathbf{T}^0$ for some $d\mathbf{T}^0$; and $\mathbf{G}^r$ from several monetary shocks
- **Q: Can I get effects of fiscal shock for different policy rules?**

McKay-Wolf, Caravello-McKay-Wolf

Semi-structural policy analysis in the sequence space

- Simple case: Given $\mathbf{G}^r$, can feed in arbitrary real rate paths $d\mathbf{r}$

$$d\mathbf{Y} = d\mathbf{Y}^0 + \mathbf{G}^r d\mathbf{r}$$

- What if I want response with a Taylor rule, e.g. $d\mathbf{r} = \phi d\mathbf{Y}$ for $\phi > 0$?

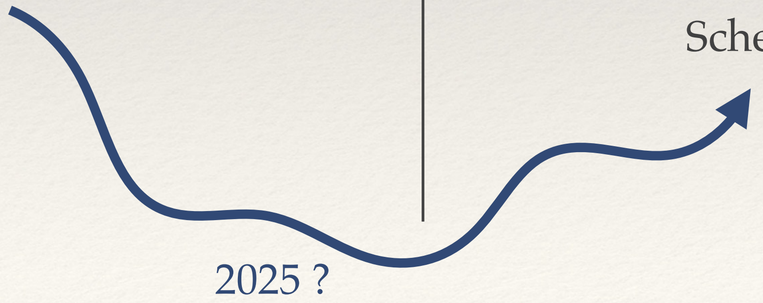
$$d\mathbf{Y} = d\mathbf{Y}^0 + \mathbf{G}^r \phi d\mathbf{Y} \quad \Longrightarrow \quad d\mathbf{Y} = \left(\mathbf{I} - \phi \mathbf{G}^r\right)^{-1} d\mathbf{Y}^0$$

- Can run monetary policy counterfactuals with $\mathbf{G}^r$!

McKay-Wolf, Caravello-McKay-Wolf

Solving heterogeneous-agent models

	Perfect foresight solution [one-time unanticipated shock at $t = 0$]		**Stochastic solution** [many stochastic shocks at all t]
Linear	Boppart-Krusell-Mitman Sequence-space Jacobian (Auclert, Bardóczy, Rognlie, Straub) SSJ in cts. time (Bilal-Goyal)	= Certainty equivalence	Reiter, Winberry Ahn et al (cts time) , Bayer-Luetticke Bilal (FAME, cts. time)
Non-linear	Various ad-hoc iterations (cts. time: Moll) Sequence-space Jacobian		Krusell-Smith, Den Haan Schaab, Fernandez-Villaverde Hurtado Nuno Scheidegger, Payne, Merkel, Maliar-Maliar Bhandari-Bourany-Evans-Golosov



SSJ code available online!

- Recent workshop materials:
  - 2023: https://www.nber.org/conferences/heterogeneous-agent-macro-workshop-spring-2023
  - 2024: https://sequence-space.com/goethe/

- Replication code for Annual Review paper: https://github.com/shade-econ/annual-review

Conclusion

Heterogeneous Agents in the Sequence Space

- HA models feature higher MPCs than RA and more persistence than TA
- Crucial for (e.g.)
  - deficit-financed fiscal policy
  - monetary policy transmission via investment
- **Natural to analyze in the sequence space!**
  - decompositions, info frictions, Bayesian estimation, FTPL, tractable alternatives, …

Recent papers using sequence space methods

- **Macro-labor**: Birinci-Karahan-Mercan-See, Faia-Shabalina-Wiczer
- **Fiscal policy / FTPL**: Angeletos-Lian-Wolf, Aguiar-Amador-Arellano
- **Optimal policy**: Davila-Schaab, McKay-Wolf, A-Cai-R-S, Yang
- **Deviations from FIRE**: Guerreiro, Rigato, Bardóczy-Guerreiro, McKay-Wieland, Wu
- **Semi-structural policy analysis**: McKay-Wolf
- **Open economy**: A-R-Souchier-S, Aggarwal-A-R-S, A-Monnery-R-S, Bellifemine-Couturier-Jamilov, Druedahl et al, Hong, Fukui-Nakamura-Steinsson
- **HANK meets finance**: Chiang-Zoch, A-R-S-Tapak, A-R-S-Wu
- **Menu cost pricing models**: A-Rigato-R-S