A Theory of Foreign Exchange Interventions*

Sebastián Fanelli CEMFI

Ludwig Straub Harvard

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Abstract

We study a real small open economy with two key ingredients: (i) partial segmentation of home and foreign bond markets and (ii) a pecuniary externality that makes the real exchange rate excessively volatile in response to capital flows. Partial segmentation implies that, by intervening in the bond markets, the central bank can affect the exchange rate and the spread between home- and foreign-bond yields. Such interventions allow the central bank to address the pecuniary externality, but they are also costly, as foreigners make carry-trade profits. We analytically characterize the optimal intervention policy that solves this trade-off: (a) the optimal policy leans against the wind, stabilizing the exchange rate; (b) it involves smooth spreads but allows exchange rates to jump; (c) it partly relies on "forward guidance", with non-zero interventions even after the shock has subsided; (d) it requires credibility, in that central banks do not intervene without commitment. Finally, we shed light on the global consequences of widespread interventions, using a multi-country extension of our model. We find that, left to themselves, countries over-accumulate reserves, reducing welfare and leading to inefficiently low world interest rates.

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1 Introduction

Volatile capital flows present complex trade-offs to central banks around the world. Chief among them is Mundell's famous trilemma, which in economies with open capital accounts manifests itself in a tough trade-off between employment and exchange rate stabilization. In search of solutions, the vast majority of open economies has turned to foreign exchange (FX) interventions. Traditionally a popular tool mostly among developing countries—nine out of ten rely on it[1]—it has recently emerged as the first line of defense among several advanced economies as well, e.g. among Euro neighbors. Across the board, intervening central banks often do not hesitate to put large sums on the line. Brazil, in the wake of the "taper tantrum", sold around US$ 110 billion worth of reserves within two years; Switzerland, fending off appreciation, has accumulated US$ 800 billion worth of reserves since 2010.

Despite the popularity of FX interventions in practice, there is little theoretical work to guide their implementation. Scant convincing empirical evidence on their effectiveness, and an influential irrelevance result by Backus and Kehoe (1989), shaped a pessimistic view within academia. In recent years, however, this view has started to shift. Serious empirical work has demonstrated their effectiveness (e.g. Kearns and Rigobon, 2005); financial frictions in capital mobility have emerged as a central ingredient in explaining puzzles at the core of international macroeconomics (Gabaix and Maggiori, 2015); and normative aspects of FX interventions have begun to move into the sights of academic economists (Liu and Spiegel, 2015; Cavallino, 2019).

Yet, many fundamental questions remain open. What kind of inefficiencies should be addressed with FX interventions? How should costs of interventions be measured? How should interventions be implemented over time? Are optimal FX interventions time consistent? What are the implications of the increasingly widespread usage of FX interventions for the world economy?

In this paper, we propose a tractable and microfounded framework that speaks to these questions. It rests on two key ingredients, the first of which is limited capital mobility. In our framework, intermediation between home and foreign bond markets is restricted due to a fixed transaction cost and position limits. These restrictions imply that intermediaries cannot arbitrage away all return differentials between markets. A portfolio balance channel thus emerges: changes in the portfolio of the central bank induce short-lived interest rate spreads between domestic and foreign bonds, as in Kouri (1976), Branson and Henderson

[1]See Canales-Kriljenko (2003).

(1985) and, more recently, Gabaix and Maggiori (2015). Thus, FX interventions are *effective*.

The second ingredient is an inefficiency in the competitive equilibrium that makes the path of the exchange rate suboptimal absent interventions. Inspired by Cravino and Levchenko (2017), this occurs in our baseline model due to a novel pecuniary externality: rich households do not take into account the effect of their spending decisions on the purchasing power of the poor via the real exchange rate. More precisely, we build a real small open economy model with two types of households: one of them is rich and Ricardian, while the other is poor and hand-to-mouth. Households have Stone-Geary preferences with subsistence needs in tradable goods, rationalizing the fact that tradable expenditure shares decrease with income (Cravino and Levchenko, 2017). When the world interest rate increases, the rich cut back their consumption, depreciating the exchange rate, and hurting the poor.[2] FX interventions can be used to affect home interest rates, thereby altering rich households' spending behavior and exchange rates. Thus, FX interventions are *desirable*.

We analyze this model through the lens of the small open economy's central bank as the social planner and ask: how should it optimally manage its holdings of foreign bonds? Our first contribution is to show that the problem can be entirely framed in terms of the *interest rate spread* that the central bank's portfolio choice generates. The logic is straightforward: to depreciate the exchange rate, the central bank sells home bonds and purchases foreign ones, generating a positive interest rate spread. Crucial to our analysis, interest rate spreads are inherently costly, over and above the standard costs from distorting (home) households' consumption profiles. The reason is that interest rate spreads invite foreign intermediaries to make profits from carry trades, representing costs to the country. These additional costs are naturally convex in the level of the spread—as more foreign intermediaries become active carry traders when spreads are higher—and increasing in the openness of the capital account—as foreign intermediaries then find it easier to take larger positions.

Our second contribution is a full characterization of the optimal FX intervention policy. We summarize our findings in six main insights or "principles". First, FX interventions should lean against the wind of global capital flows, dampening exchange rate movements. In our baseline model, this is desirable because it stabilizes the welfare of the poor.[3] Second,

[2]This model is meant to capture a significant concern of policymakers in emerging markets: the pass-through of exchange rate movements into food prices - an important component of the consumption basket of low income households. However, our results are much more general than this particular setup. We present a general framework in Section 4.1 and three alternative applications in Appendix D.

[3]In other applications, studied in Appendix D, leaning against the wind is desirable for different reasons. For example, in an environment with aggregate demand externalities, leaning against the wind incentivizes agents to spend more during recessions, which increases welfare. In a model with endogenous terms of trade

FX interventions should be smooth in terms of interest rate spreads. This helps spread out the costs from FX interventions over time, reminiscent of the famous tax-smoothing model of Barro (1979). Notably, our cost term is *not* equal to a simple quasi-fiscal cost, which is the standard cost measure used in the literature (Adler and Mano, 2018). In fact, in our calibration, it is one order of magnitude smaller as it takes into account the financial benefits directly accruing to home households. We demonstrate that our idea of FX intervention smoothing does not imply that the exchange rate path should be smooth. That is, the exchange rate must be allowed to "jump" in response to the shock. A price-based policy (e.g. a "crawling peg") that tries to slow the adjustment over time necessitates large interest-rate spreads and thus carries large welfare costs. Our third insight is a direct consequence of FX-intervention smoothing, namely that the optimal policy involves promises of future FX interventions even when the actual shock has already passed—a form of "FX forward guidance".

Our fourth insight is that these promises naturally lead to a new time inconsistency problem, as central banks would renege on their promised FX interventions after the shock has subsided. When FX interventions cannot be credibly promised at all, we show that *no* FX intervention will be chosen in the associated time-consistent Markov equilibrium, highlighting the role of central bank credibility as an essential input into successful conduct of FX intervention policies. Our fifth insight echoes the common perception that it is "easier" to resist appreciation than to fight depreciation. We rationalize this on the basis that households have asymmetric access to foreign markets. Simply put, if households can save in dollars more than they can borrow, the central bank needs to commit more resources to defend the currency against depreciation than against appreciation. Finally, for our sixth insight, we consider a "non-fundamental" shock, such as the noise shocks in Gabaix and Maggiori (2015). In contrast with fundamental capital flow shocks, we argue that these shocks are not costly and instead lead to financial gains for the intervening country.

We offer a rich set of extensions in Section 4 and Appendix D. Among others, we propose a general framework that can nest alternative motives of interventions and show the robustness of our results; we introduce long-term assets in the economy and prove that our time inconsistency cannot be mended by maturity management as in Lucas and Stokey (1983); and, finally, we study whether our interest rate spreads correspond to uncovered- or covered-interest-rate-parity violations.

Our last contribution is to characterize the positive and normative consequences of

and home bias, curbing exchange rate movements helps increase the price of exports when net exports are large. We identify the common thread across applications in our general framework of Section 4.1.

widespread FX interventions for the international monetary system. We embed our baseline model in a world composed of a continuum of small open economies, which are subject to limited capital mobility as before. We simulate a global savings glut by symmetrically increasing all households' desires to save, capturing recent trends like population aging or a growth slowdown. We show that in response to other countries' savings behavior, each country finds it individually optimal to engage in FX interventions and accumulate reserves. Yet, this only "pushes" more savings into the other countries, which amplifies their desired interventions in the Nash equilibrium, resulting in "reserve wars". We find that reserve wars are characterized by large *public* cross-border capital flows (as in Aguiar and Amador, 2011 and Gourinchas and Jeanne, 2013) and by significantly depressed world interest rates. Strikingly, a planner that could coordinate all countries' interventions would optimally ban their usage in response to symmetric shocks (such as a global savings glut), and only allow them in response to country-specific asymmetric shocks, where they have insurance benefits.

Literature. Our paper builds on a recent literature studying FX interventions using fully microfounded frameworks with limited capital mobility.[4] Our financial friction is, in reduced form, the same as in Gabaix and Maggiori (2015). They introduce it in a general equilibrium environment and illustrate that FX interventions can be effective in moving exchange rates. Chang and Velasco (2017) study FX interventions in an environment with borrowing constraints, and show they are effective when these constraints are binding. Liu and Spiegel (2015) numerically solve for the jointly optimal response of taxes on financial assets, FX interventions, and monetary policy to fundamental shocks in a New-Keynesian model, finding that FX interventions lean against the wind. Cavallino (2019) also uses a New-Keynesian model and studies optimal FX interventions against non-fundamental capital inflow shocks, characterizing the solution to first-order around the steady state. Amador et al. (2020) characterize the FX interventions that are necessary to sustain a given exchange rate path in an environment with a zero lower bound on nominal interest rates. They also measure the cost of FX interventions in Switzerland as the product of the covered-interest-rate-parity (CIP) deviation and the stock of reserves. Relative to these papers, we contribute on the positive side by deriving an expression for the costs of FX interventions, emphasizing the residence of home-bond holders and the maturity structure. On the normative side, we contribute by proposing new motives for interventions and by identifying several new principles.

[4] Ostry et al. (2012), Benes et al. (2015), Devereux and Yetman (2014), and Blanchard et al. (2014) study the effects of interventions without a fully microfounded model.

Our paper is also connected to the burgeoning literature on optimal capital controls, which also characterizes optimal paths of tax-induced interest rate spreads.[5] Indeed, two of our alternative models share common themes with Farhi and Werning (2012, 2014) but allow for limited capital mobility. We show that their prescription that the optimal policy should lean against the wind after world interest rate shocks, stabilizing movements in the real exchange rate, carries over to our environment. However, none of the other principles is present in their optimal capital controls problem.

Our paper is also related to a large literature studying the long-drawn process of reserve accumulation by emerging market central banks in the past decades.[6] Closest to us in this literature are Benigno and Fornaro (2012), Jeanne (2012) and Bacchetta et al. (2013, 2014), which investigate the role of reserve accumulation in economies in which the private sector lacks access to foreign markets. However, in these models there is no region in which the planner balances the benefits of FX interventions with costs (beyond distorting consumption)—which is at the center of our analysis.

Finally, our study of a world equilibrium with reserve accumulation in Section 5 is related to Obstfeld (2013), who emphasizes the dangers of currency wars through reserve accumulation and its consequences for global interest rates. Models of low global interest rates are also put forth by Coeurdacier et al. (2015) and Caballero and Farhi (2018). We contribute to this literature by showing how decentralized FX interventions can be a powerful amplification mechanism of an initial rise in global savings.

Layout. The paper is organized as follows. In Section 2, we present our baseline model and derive the planning problem. In Section 3, we characterize the optimal policy, organizing the exposition around six main insights or "principles". In Section 4, we develop a general framework that nests the baseline model and use it to show the robustness of our results. We also extend our baseline model to include long-term assets and currency forwards. In Section 5, we present a multi-country version of our model. Section 6 concludes. The appendix contains all proofs, as well as additional extensions and details on the calibration and data used in the paper.

[5] See, among many others, Bianchi (2011); Magud et al. (2018); Farhi and Werning (2012, 2014); Costinot et al. (2014); Heathcote and Perri (2016).

[6] See, for instance, Aizenman and Lee (2007); Alfaro and Kanczuk (2009); Korinek and Serven (2016); Jeanne and Rancière (2011); Benigno and Fornaro (2012); Bianchi et al. (2018); Hur and Kondo (2016); Jeanne (2012); Gopinath and Stein (2018).

2 Baseline Model

In this section, we focus on a model inspired by the work of Cravino and Levchenko (2017). We study an abstract general framework that nests this model as a special case in Section 4.1. We consider alternative applications of the general framework in Appendix D.

2.1 Model setup

We study a real small open economy (SOE) model in continuous time. There are three kinds of agents: the SOE's households, its central bank, and financial intermediaries. The agents interact in two asset markets, one for the SOE's home bonds, and one for foreign bonds. We describe all three kinds of agents in turn.

Households and goods markets. There are two types of households $i \in \{R, P\}$ in the home country—a mass $1-\mu$ *Ricardian*, or *rich*, ($i = R$) and a mass μ *poor* ($i = P$) households. Households trade and consume tradable and nontradable goods, of which the SOE's total endowments are y_T and y_N. Each household maximizes a Stone-Geary utility function

$$\int_0^\infty e^{-\rho t} u\left((c^i_{Tt} - \underline{c})^\alpha (c^i_{Nt})^{1-\alpha}\right) dt \tag{1}$$

where c^i_{Tt} is tradable consumption and c^i_{Nt} is nontradable consumption of a household of type i; α represents the degree of openness; $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$ is a CES flow utility function; and $\underline{c}$ represents a subsistence level in tradable goods consumption (e.g. food). The latter introduces a non-homotheticity in consumption, which captures the fact that poor households spend a higher fraction of their income on tradables, and within tradables, on goods with systematically lower nontradable components (Cravino and Levchenko, 2017). As a result, the cost of poor households' consumption baskets is particularly sensitive to exchange rate fluctuations, which, as we discuss at great length below, has immediate consequences for central bank policies.

Ricardian and poor households differ in two aspects: (i) their income stream, and (ii) their access to financial markets. Ricardian households own a share $1 - \chi > 1 - \mu$ of the SOE's endowments, have unfettered access to home financial markets, and some restricted access to international financial markets. Poor households own a share $\chi < \mu$ of endowments in the economy and lack access to any financial markets.

Throughout our analysis, we normalize the foreign good's price to 1 and refer to that numeraire as "dollars". The consolidated dollar budget constraint of all Ricardian households is then

$$p_t c^R_{Nt} + c^R_{Tt} + \dot{b}_{Ht} + \dot{b}^*_{Ht} = y_T + r_t b_{Ht} + r^*_t b^*_{Ht} + t_t + \pi_t, \tag{2}$$

where p_t is the relative price of the nontradable good, b_{Ht} and b^*_{Ht} are the households' positions in home and foreign bonds, t_t are transfers to or from the central bank, π_t are intermediation profits (to be specified below), and r_t and r^*_t are the returns of the home and foreign bond, respectively (both in dollars).[7] p_t can be regarded as the inverse of the exchange rate, which, up to a constant factor, is equal to $p_t^{-(1-\alpha)}$. The position in foreign bond markets is assumed to be restricted,

$$b^*_{Ht} \in [\underline{b}^*_H, \overline{b}^*_H]$$

where $\underline{b}^*_H \leq 0 \leq \overline{b}^*_H$, and $|\underline{b}^*_H| \leq \overline{b}^*_H$.[8] This assumption captures the idea that it is difficult for many households in emerging markets to frictionlessly access international financial instruments without having to rely on financial intermediaries, especially so when borrowing abroad. This specification allows as special cases the commonly assumed case where households cannot access financial markets at all without intermediaries, $\underline{b}^*_H = \overline{b}^*_H = 0$, as well as the case where $\overline{b}^*_H$ is significantly larger than $|\underline{b}^*_H|$ and thus access is asymmetric.

For a poor household, the dollar budget constraint is given by

$$\mu z^P_t = \chi p_t y_N + \chi y_T - \mu \underline{c}. \tag{3}$$

where $z^i_t \equiv c^i_{Tt} - \underline{c} + p_t c^i_{Nt}$ denotes the (net-of-subsistence) dollar expenditure by a household of type i. When $\mu \underline{c} > \chi y_T$, the poor's endowment of the tradable good is insufficient to cover their subsistence needs. Thus, a depreciation forces them to cut their non-subsistence spending significantly in order to finance their subsistence needs. Formally, z^P_t moves more than proportionally with p_t. Henceforth, we assume this is the case, in line with the idea that devaluations particularly hurt the poor.

[7] Since the model is deterministic, the currency denomination of the returns is irrelevant when the country has zero net liabilities in the home currency at $t = 0$. We allow for non-zero home currency liabilities in Section 4.3.

[8] Although there are restrictions on holdings of foreign bonds, we refer to these households as Ricardian because the timing of lump-sum transfers is irrelevant to them.

The optimal demands for tradable and nontradable goods are given by

$$c_{Tt}^{i} = \alpha z_t^i + \underline{c} \tag{4}$$
$$c_{Nt}^{i} = p_t^{-1}(1-\alpha) z_t^i \tag{5}$$

Below, we often write z_t instead of z_t^R, as it will play a prominent role in the analysis. Unlike poor households, Ricardian households have unfettered access to home bond markets. Thus, their total expenditure z_t satisfies the following Euler equation,

$$\frac{\dot{z}_t}{z_t} = \frac{1}{\sigma}(r_t - \rho) + (1-\alpha)\frac{\sigma - 1}{\sigma}\frac{\dot{p}_t}{p_t}. \tag{6}$$

Their demand for foreign bonds is at the upper bound, $b_{Ht}^* = \overline{b}_H^*$, when $r_t^* > r_t$, and vice versa at the lower bound, $b_{Ht}^* = \underline{b}_H^*$, when $r_t^* < r_t$. When $r_t = r_t^*$, b_{Ht}^* is indeterminate.

Financial intermediaries. The key ingredient in our model that makes FX interventions effective is a finite elasticity of the demand for home bonds. As a result, a change in the portfolio of the central bank affects the return of home assets r_t relative to its foreign counterpart r_t^*, henceforth referred to as the *interest rate spread*. Backus and Kehoe (1989) pointed out that these portfolio balance effects are muted in general equilibrium in a world with free movement of capital, as the private sector would perfectly undo any actions by the central bank.

We break this result by modeling limited asset market participation, in the spirit of Bacchetta and Van Wincoop (2010) and Gabaix and Maggiori (2015). In particular, we assume that there exists a continuum of intermediaries owned by foreigners, labeled by $j \in [0, \infty)$, which can trade in both foreign and home bond markets.

Foreign intermediaries' investment decisions are subject to two important restrictions. First, each intermediary is subject to a net open position limit $X > 0$.[9] Second, we follow Alvarez et al. (2009) in assuming that intermediaries face heterogeneous participation costs. In particular, each intermediary j active in the home bond market at time t is obliged to pay a participation cost of exactly j per dollar invested.

Putting these ingredients together, intermediary j optimally invests an amount x_{jt},

[9] It is worth noting that many emerging market central banks do impose position limits on intermediaries' investments as a form of capital controls, hence artificially decreasing X, see e.g. Canales-Kriljenko (2003).

solving

$$\max_{x_{jt}\in[-X,X]} x_{jt}\left(r_t - r_t^*\right) - j|x_{jt}|.$$

Intermediary j's cash flow conditional on investing is $X\left|r_t - r_t^*\right|$ while participation costs are jX. Thus, investing is optimal for all intermediaries $j \in [0, \bar{j}]$, with the marginal active intermediary $\bar{j}$ given by $\bar{j} = |r_t - r_t^*|$. The aggregate investment volume is then

$$b_{It} = \bar{j}X \cdot \text{sign}\left(r_t - r_t^*\right).$$

Defining $\Gamma \equiv X^{-1}$ and substituting out $\bar{j}$, we obtain

$$b_{It} = \frac{1}{\Gamma}\left(r_t - r_t^*\right). \tag{7}$$

Equation (7) embodies that foreign intermediaries' demand for home bonds has a finite (semi-)elasticity to the return spread. This equation is crucial to our analysis because it implies that changes in home bond demand, e.g. induced by FX interventions, can indeed affect home interest rates.

The critical parameter in (7) is the inverse demand elasticity Γ. If Γ is large, e.g. due to tight position limits X, intermediation is impeded. In equilibrium, this implies both small levels of b_{It} and a small sensitivity of b_{It} to the interest rate spread. In the extreme case where $\Gamma \to \infty$, foreign intermediation is absent, $b_{It} = 0$, and home households have no access to foreign investments beyond their own. By contrast, if Γ is small, e.g. due to relaxed position limits X, the equilibrium will feature both large b_{It} and a large sensitivity of b_{It} to the interest rate spread. In the extreme case where $\Gamma \to 0$, bond demand adjusts so that $r_t = r_t^*$ and the elasticity is infinite. Henceforth, we assume $\Gamma \in (0, \infty)$.

We also allow for intermediaries owned by households in the home country. Similar to their foreign counterparts, home intermediaries' optimal home bond position is given by

$$b_{IHt} = \frac{1}{\Gamma_H}\left[r_t - r_t^*\right] \tag{8}$$

where $\Gamma_H \in (0, \infty)$ is the inverse demand elasticity.[10] Profits generated by home intermediaries, $\pi_t \equiv b_{IHt}\left[r_t - r_t^*\right]$, are paid to Ricardian households.

[10]We assume that participation costs constitute transfers to Ricardian agents in the home economy. Thus, no extra cost terms enter the budget constraint (2).

Central bank. The home central bank acts as the home country's social planner in our model. It chooses a foreign exchange intervention policy $\{b_{Gt}, b^*_{Gt}, t_t\}$ consisting of home bond investments b_{Gt}, foreign bond investments b^*_{Gt}, and transfers t_t to Ricardian households, subject to the central bank budget constraint[11]

$$\dot{b_{Gt}} + \dot{b^*_{Gt}} = r_t b_{Gt} + r^*_t b^*_{Gt} - t_t. \tag{9}$$

The central bank's interventions must also ensure that the country satisfies a no-Ponzi condition,

$$\lim_{t\to\infty} e^{-\int_0^t r^*_s ds} \mathrm{nfa}_t = 0 \tag{10}$$

where $\mathrm{nfa}_t \equiv b_{Ht} + b^*_{Ht} + b_{Gt} + b^*_{Gt}$ is the net foreign asset position of the country. Observe that in this economy, it is without loss to set $b^*_{Gt} + b_{Gt} = 0$ due to the availability of transfers between the central bank and Ricardian households.

Competitive equilibrium. The model is closed with the goods market clearing condition in nontradables,

$$(1-\mu)c^R_{Nt} + \mu c^P_{Nt} = y_N \tag{11}$$

and the home bond market clearing condition,

$$b_{Ht} + b_{It} + b_{IHt} + b_{Gt} = 0. \tag{12}$$

We formally define the competitive equilibrium in this environment as follows.

Definition 1. Given initial debt positions $(b_{H0}, b_{I0}, b_{IH0}, b_{G0}, b^*_{G0})$, a path for the international interest rate $\{r^*_t\}$, and a central bank FX intervention policy $\{b_{Gt}, b^*_{Gt}, t_t\}$, an allocation $\{c^i_{Nt}, c^i_{Tt}, b_{Ht}, b^*_{Ht}, b_{It}, b_{IHt}, \pi_t\}$ together with prices $\{p_t, r_t\}$ is a *competitive equilibrium* iff they jointly solve (2)–(12).

We assume that the aggregate tradable endowment y_T is sufficiently large to guarantee the existence of a competitive equilibrium.

[11] We implicitly assume that the relevant interest rate for marginal changes in reserves is r^*_t. One might argue that negative levels of b^*_{Gt} are associated with a different, higher interest rate. In reality, however, reserves are (almost) always positive. Thus, marginal changes in reserves are associated with the foreign interest rate on savings, r^*_t.

2.2 Equilibrium characterization and implementability

Next, we characterize the competitive equilibrium, with the goal to derive "implementability conditions" describing the set of competitive equilibria that can be attained by different FX intervention policies. Substituting nontradable consumption demands (4) into the goods market clearing condition (11) gives us an expression for the price of nontradable goods,

$$p_t = p(z_t) \equiv y_N^{-1} \frac{1-\alpha}{1-\chi(1-\alpha)}((1-\mu)z_t + \chi y_T - \mu \underline{c}). \tag{13}$$

This equation shows that, when the expenditure z_t of Ricardian agents increases by 1%, the price of nontradables rises by more than 1%. Together with (3), this implies that poor households' expenditures are more volatile than Ricardian households' expenditures in the competitive equilibrium. Is this efficient? Replacing consumption demands (4) and (5) into flow utility (1), it follows that households have identical preferences in terms of z_t^i. Thus, expenditures are equally volatile in the first best, i.e. an environment where poor households can also access financial markets. This already suggests that, in a constrained efficient allocation, the planner may be willing to reduce the volatility of the expenditure path of the Ricardian agents to smooth the expenditure path of the poor.[12]

Using the Ricardian households' consolidated dollar budget constraint (2), we obtain

$$\frac{\alpha}{1-\alpha} p(z_t) y_N + \underline{c} + \dot{b}_{Ht} + \dot{b}_{Ht}^* = y_T + r_t b_{Ht} + r_t^* b_{Ht}^* + t_t + \pi_t, \tag{14}$$

where $\frac{\alpha}{1-\alpha} p(z_t) y_N + \underline{c}$ is the total consumption of tradable goods. In (14), variables t_t and π_t can be eliminated by adding the central bank's budget constraint (9) as well as the expression of home intermediaries' profits. This allows us to rewrite the households' budget constraint as a country-wide budget constraint,

$$\dot{\text{nfa}}_t = y_T - \frac{\alpha}{1-\alpha} p(z_t) y_N - \underline{c} + (r_t - r_t^*)(b_{Ht} + b_{Gt} + b_{IHt}) + r_t^* \text{nfa}_t. \tag{15}$$

In this equation, policy variable b_{Gt} can be expressed as $-b_{Ht} - b_{It} - b_{IHt}$ using home bond market clearing (12), where intermediaries' bond demand b_{It} is given by (7). After this

[12] Note that we have not resorted to arguments based on *ex ante* distributional considerations, which cannot be improved upon by intertemporal tools such as FX interventions (see Costinot et al., 2014 for a similar remark in the context of capital controls). Indeed, in a stationary environment, the planner would not use FX interventions whatever her distributional objectives may be—a corollary of Proposition 2.

substitution, the country-wide budget constraint (15) simplifies to

$$\dot{\text{nfa}}_t = \underbrace{y_T - \frac{\alpha}{1-\alpha} p(z_t) y_N - \underline{c}}_{\text{net exports}} + \underbrace{r_t^* \text{nfa}_t}_{\text{interest income}} - \underbrace{\frac{1}{\Gamma} \left(r_t - r_t^*\right)^2}_{\text{costs from interest rate spreads}} . \tag{16}$$

Up to the last term, equation (16) is a standard open economy budget constraint. It states that home's net foreign asset position improves if the net exports are large or if the country is a creditor. By contrast, the last term is novel. It captures the costs the country incurs if the interest rate spread $r_t - r_t^*$ is different from zero.

Why does the country face costs from nonzero interest rate spreads? Suppose the spread $r_t - r_t^*$ is positive. This invites foreign intermediaries to enter the home bond market and take a position $b_{It} = \frac{1}{\Gamma}\left(r_t - r_t^*\right)$. As a result, they earn

$$b_{It} \cdot \left(r_t - r_t^*\right) = \frac{1}{\Gamma} \left(r_t - r_t^*\right)^2 . \tag{17}$$

These carry trade profits are an economic cost to the SOE, which is taking the other side of the carry trade. These costs increase when foreign intermediaries' demand becomes more elastic (low Γ), since intermediaries take larger positions for any given interest rate spread. Crucially, these costs are independent of home agents' access to financial markets (independent of Γ_H, $\underline{b}_H^*$ and $\overline{b}_H^*$). The reason is that any profits earned by home intermediaries are paid to Ricardian households at home, leaving the total wealth of the country unaltered.

Next, we study the set of all equilibria that are implementable by FX interventions. For this result and the remainder of the paper, we introduce as notation for the interest rate spread $\tau_t \equiv r_t - r_t^*$. Rewriting the budget constraint (16) in present value terms we obtain the following implementability result.

Proposition 1 (Implementability conditions.)**.** *Let $\tau_t = r_t - r_t^*$ be the spread between home and foreign interest rates. Then, given an initial net foreign asset position nfa_0 and a path for the international interest rate $\{r_t^*\}$, the paths $\{c_{Tt}^R, c_{Nt}^R, c_{Tt}^P, c_{Nt}^P\}$ and $\{p_t, r_t\}$ are part of a competitive equilibrium iff the corresponding $\{z_t, \tau_t\}$ solve the following two conditions: the Euler equation,*

$$\Sigma(z_t) \frac{\dot{z}_t}{z_t} = r_t^* + \tau_t - \rho \tag{18a}$$

where $\Sigma(z) \equiv \sigma + (1-\alpha)(1-\sigma)\frac{p'(z_t)z_t}{p(z_t)}$ *and the country-wide present value budget constraint,*[13]

$$\int_0^\infty e^{-\int_0^t r_s^* ds}\left[\frac{\alpha}{1-\alpha}p(z_t)y_N + \underline{c} - y_T + \frac{1}{\Gamma}\tau_t^2\right]dt \quad = \quad \text{nfa}_0. \tag{18b}$$

Proposition 1 gives us a simple characterization of the set of competitive equilibria as it is commonly used in models of optimal Ramsey taxation (see, e.g. Lucas and Stokey, 1983 or Chari and Kehoe, 1999). A key difference with this literature is that the planner in our model does not choose a path of taxes, but rather an FX intervention policy as defined above. Proposition 1 is important because it implies that setting interventions—which are paths of asset positions—is equivalent to setting interest rate spreads τ_t—which behave like taxes.

Having described the set of implementable allocations, we next turn to the full planning problem.

2.3 Planning problem

We consider the problem of a utilitarian planner putting equal weight on each type of household, and we envision the central bank as fulfilling this role. The planner's problem is, therefore, to maximize the sum of home households' welfare by choosing among the competitive equilibria it can implement using FX interventions.

Using equations (3)-(5) and (11), we can state the planning problem as[14]

$$\max_{\{z_t,\tau_t\}} \int_0^\infty e^{-\rho t} V(z_t) dt \tag{19}$$

where

$$V(z) = p(z)^{-(1-\alpha)(1-\sigma)}\left\{(1-\mu)u(z) + \mu u\left(p(z)\chi\mu^{-1}y_N + \chi\mu^{-1}y_T - \underline{c}\right)\right\} \tag{20}$$

subject to the two implementability conditions (18a) and (18b).

In the planning problem (19), the freedom of setting different FX intervention policies is completely embodied in the choice of the interest rate spread τ_t. When the central bank desires to raise consumption in period t relative to the next, it lowers τ_t. Such a policy would then be implemented by selling reserves and purchasing home bonds, which, due

[13]We assume $\sigma + (1-\sigma)\frac{1-\alpha}{1-\chi(1-\alpha)} > 0$ to ensure $\Sigma(z) > 0$ (see Appendix B.2). This condition is satisfied in our calibration.

[14]See Appendix B.2 for a derivation and a proof that $V(z)$ is strictly increasing and concave.

to a finitely elastic foreign demand function, affects the home interest rate r_t and thus τ_t.

One possibility for the central bank in this baseline model is to set $\tau_t = 0$ in all periods. This is feasible because the central bank has unrestricted access to both bond markets. If home households have sufficient access to foreign markets, i.e. $\underline{b}^*_H$ and $\overline{b}^*_H$ are sufficiently large, no intervention is required to implement $\tau_t = 0$. By contrast, if home households have minimal access, then there is a clear role for the central bank to step in and do the intermediation itself, taking positions to ensure $r_t = r^*_t$. While this may be realistic in certain situations of sudden illiquidity in FX markets, we henceforth focus our attention on the more interesting case where the financial friction does not affect the competitive equilibrium, i.e. when home households have enough access such that $\tau_t = 0$ is feasible absent any interventions.

2.4 Discussion

We next discuss the main assumptions behind our model, as well as alternative tools the planner might have access to.

Main ingredients. Our model has two key ingredients. The first ingredient is an inefficiency in the competitive equilibrium allocation, providing the planner with a motive for intervening. In our baseline model, the source of inefficiency is a pecuniary externality: Ricardian households do not take into account that their spending decision affects the income of the poor. Indeed, as we argued before, the expenditure path of Ricardian agents is too volatile in the competitive equilibrium (we show this formally in Section 3). We further demonstrate in Section 4.1 that similar motives for intervening are implied by alternative sources of inefficiency, such as terms-of-trade manipulation and aggregate demand management. One way to see this already now is that the planning problem solely depends on the planner's objective as a function of home agents spending, $V(z)$, and the inverse elasticity of private spending to the interest rate, $\Sigma(z)$. In Section 4.1, we identify the crucial properties of these two objects that explain the robustness of our results across applications.

The second ingredient is limited capital mobility, providing the planner with the ability to affect allocations through interventions. This works because the private sector is at a disadvantage to undo its actions: both households and arbitrageurs face position limits. Our microfoundation of limited capital mobility rests on two main components. The first is rationality, that is, agents only participate if they make profits; the second is the property that participation increases in the size of the spread.[15] These two assumptions have important implications: going "against the market" entails costs for the government; and costs increase more than proportionally with the intervention size, as more agents are encouraged to exploit the arbitrage opportunity.

In sum, the first ingredient gives the planner a motive to intervene, while the second implies interventions are effective, but increasingly costly. These features are at the core of our characterization of optimal policy in Section 3.

Alternative tools. In our main model, the planner cares about the distribution of wealth across types. One may think that this suggests that the most appropriate tool to address this problem are transfers. Indeed, a full set of agent-specific *and* time-varying transfers can trivially attain the first best by replicating the path of desired asset positions of the poor. In realistic settings, however, such a rich set of tools is likely unavailable.[16] Observe that time-invariant transfers are not sufficient. These would only allow planner to redistribute wealth on average, but cannot solve the issue of "excess" volatility of private spending present in our environment.[17]

Furthermore, given that our planner is trying to affect the path of private spending over time, one may wonder if taxes on financial flows (capital controls) are not a better and more direct policy tool. In fact, a reader that is familiar with the recent literature on capital controls (see, e.g. Bianchi (2011); Farhi and Werning (2012, 2014); Jeanne (2012); Heathcote and Perri (2016)) may notice that after taking the limit $\Gamma \to \infty$ the planning problem (19) becomes formally equivalent to the archetypical optimal capital controls problem: The planner directly controls the wedge between between r_t and r_t^* at a zero resource cost. One must be careful, however, with the economic interpretation of this limit. If the private sector is in financial autarky, which is precisely the case when $\Gamma \to \infty$ and $\underline{b}_H^* = \bar{b}_H^* = 0$, taxes are non-allocative, so the only implementable allocation is the one in which trade is

[15] We assumed above that participation increases linearly with the spread to simplify the exposition. We generalize to arbitrary nonlinear increasing demand schedules $b_{It} = g(r_t - r_t^*)$ in Section D.1.

[16] Second-best or third-best environments like ours are motivated by the challenges associated with implementing these ideal policy tools. For example, it is a well-understood problem in public finance that the government faces a severe incomplete information problem when devising its tax policies (Mirrlees, 1971). In addition, fiscal policy is typically far too slow to respond to business-cycle-frequency shocks and movements in exchange rates.

[17] One may also wonder whether taxes on nontradables help. The answer depends on how the proceeds from these taxes is rebated. That is, it is again a question of whether one can use them to redistribute wealth. Let τ_{Nt} denote the ad-valorem tax. If taxes on nontradables are available but the proceeds are rebated in a "neutral" manner, i.e. in proportion to the ownership of the nontradable endowment $T_t^P = \chi \tau_{Nt} p_t y_N$ and $T_t^R = (1-\chi)\tau_{Nt} p_t y_N$, they do not affect equilibrium allocations and are merely reflected in a lower nontradable price before taxes.

always balanced. By contrast, this is the parametrization that makes FX interventions most effective.

When the private sector is not in financial autarky, i.e. when $\underline{b}^*_H, \bar{b}^*_H \neq 0$ or when $\Gamma < \infty$, taxes on financial assets seem very effective: they can implement the desired wedge at a zero resource cost. However, this argument presumes that taxes are perfectly enforceable. In practice, an important concern of policymakers when deciding whether to implement taxes on financial flows is the ability of the private sector to circumvent them, i.e. whether capital controls may "leak", as in Bengui and Bianchi (2014). Explicitly modeling the ability of foreigners to avoid the taxes imposed on cross-border transactions would lead to a resource cost term in the budget constraint that is similar to the one studied in our optimal FX-intervention problem.[18]

In this sense, the planning problem studied in this paper is a third-best problem. The first best is having time-varying agent-specific taxes; the second best is controlling the wedge costlessly (either $\Gamma \to \infty$ in our setup or $\Gamma < \infty$ and perfectly enforceable taxes); the third best adds an additional resource cost related to the "undoing" activity of the private sector.

Finally, we focused the discussion on a particular capital control: taxes on inflows. In reality, capital controls include a broader array of tools, such as position limits and reserve requirements, which can effectively put "sand in the wheels" of private intermediation. In the language of our model, such capital controls may serve to increase Γ. Thus, they are complementary to FX interventions. This is in line with recent evidence showing that capital controls are usually not cyclical (Fernández et al., 2016), and that FX interventions become more effective when capital controls are in place (Kuersteiner et al., 2018). Note, however, that to the extent that Γ reflects institutional barriers such as capital controls, the private sector may learn to circumvent them. We address this concern in Appendix D.6, where we let the degree of capital market frictions Γ_t fall over time. Such a falling Γ_t precisely captures the idea that markets become more and more frictionless as an intervention persists.[19]

Table 1: Illustrative calibration (details in Appendix A).

Parameter	Symbol	Value	Calibration
Share poor households	μ	0.2	population share
Endowment share of poor	χ	0.078	expenditure share of poor
Preference weight on tradables	α	0.051	tradable expenditure shares
Subsistence level	$\underline{c}$	0.19	tradable expenditure shares
Tradable endowment	y_T	0.23	zero NFA
Nontradable endowment	y_{NT}	1	normalization
Inverse EIS	σ	2	standard value
Discount rate	ρ	0.075	5yr Treasury yield, EMBI+Brazil
Capital immobility (home)	Γ_H	3	relative balance sheet of Brazilian banks
Capital immobility (foreign)	Γ	9	Kohlscheen and Andrade (2014) intervention

3 Six Principles of Optimal FX Interventions

We are now in a position to study the planning problem (19) and distill six insights, or "principles", about optimal FX interventions in response to capital flows. We explain our principles with model simulations using an illustrative calibration based on Brazil, shown in Table 1. All details on the calibration can be found in Appendix A.

3.1 Leaning against the wind

The setting of the first five principles is that of a temporary shock to world interest rates,

$$r_t^* = \begin{cases} r^* + \frac{\Delta r^*}{T}(T-t) & t < T \\ r^* & t \geq T \end{cases} \tag{21}$$

where Δr^* and T are the size and duration of the shock, respectively. Our first principle concerns the direction of the intervention.

Proposition 2 (Leaning against the wind.). *In response to capital inflows, $\Delta r^* < 0$, optimal FX interventions require reserve accumulation, $b_{Gt}^* > 0$, thus inducing a positive interest-rate spread $\tau_t > 0$ and dampening the appreciation of the exchange rate.*

Conversely, in response to capital outflows, $\Delta r^* > 0$, optimal FX interventions require reserve

[18] The revealed preference of policy makers for FX interventions instead of time-varying capital controls, which are rarely employed in practice (Fernández et al., 2016), may reflect that these costs are perceived as higher relative to the carry-trade profits by the private sector.

[19] Recall that t denotes the time since the arrival of a shock, not calendar time.

decumulation, $b^*_{Gt} < 0$, inducing a negative interest-rate spread $\tau_t < 0$ and containing the depreciation of the exchange rate.

Proposition 2 shows that optimal FX interventions lean against the wind, accumulating reserves when capital flows into the country and vice versa. To understand this result, consider the case of a capital inflow shock, illustrated in Figure 1. The black line shows the response with laissez-faire, i.e. with $\tau_t \equiv 0$. In response to the low foreign interest rates, Ricardian households borrow abroad. This increases the demand for nontradables and leads to an appreciation of the exchange rate. The green lines show the response with optimal FX interventions for different degrees of financial frictions Γ in global capital flows, $\Gamma = 9$ being our calibrated value.

Why is the laissez-faire response suboptimal? Recall that poor households' total expenditures z^P_t are relatively more sensitive to the exchange rate than Ricardian households', z_t (see our discussion below (13)). Since preferences are identical in terms of z^i_t, z^P_t is too steep, or volatile, intertemporally under laissez-faire. By buying reserves and selling home bonds, the central bank increases the yield on the home bond, lowering the desired borrowing of Ricardian households. This depreciates the exchange rate, stabilizes the income of the poor, and partially eliminates the excess volatility in their consumption.

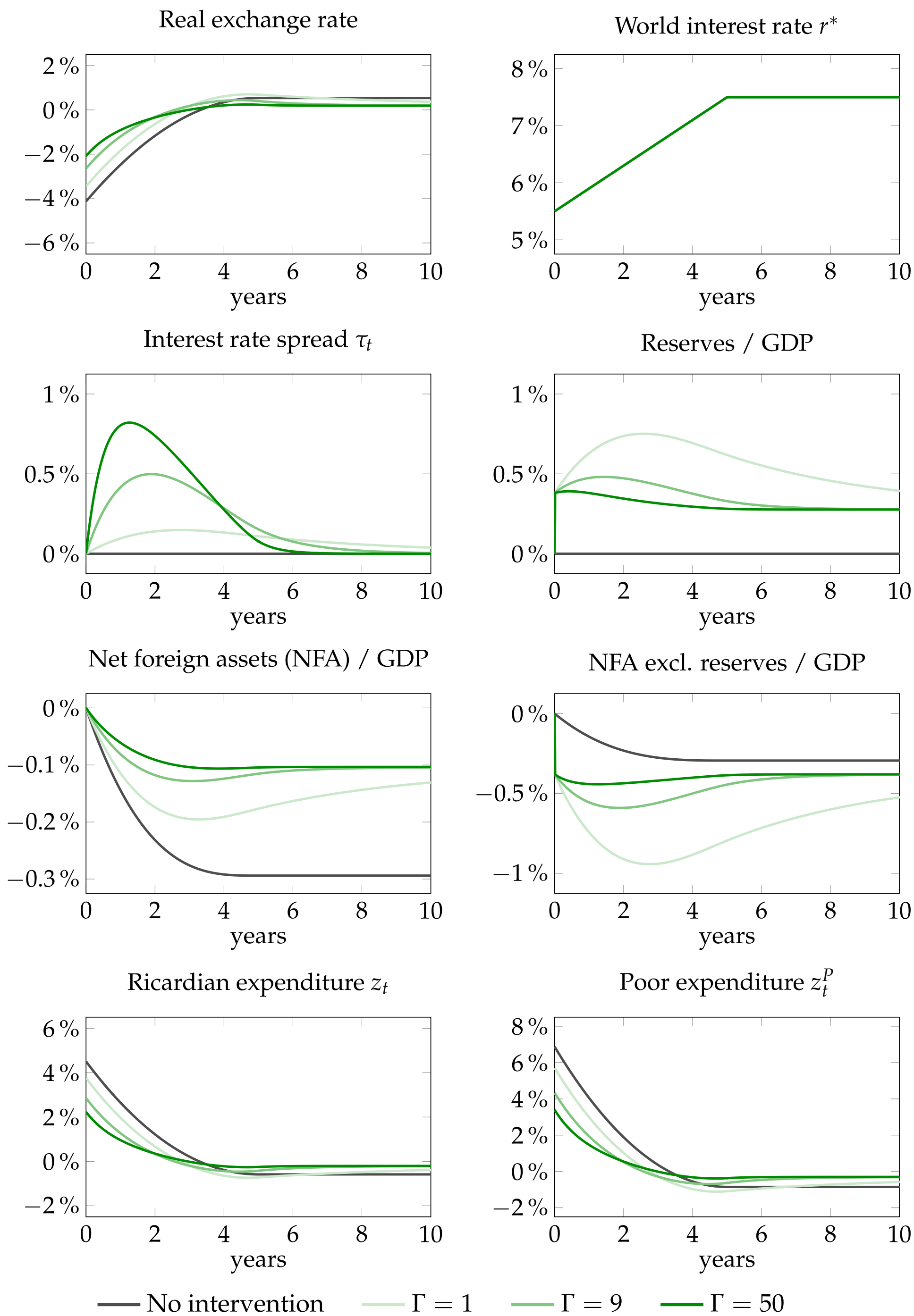
The planner thereby finds it optimal to increase reserves sharply at the onset and then continually reinvests to keep them persistently high. The sharp jump is necessary since the central bank needs to undo the frictionless borrowing from home households (i.e. up to $\underline{b}^*_H$) to have an effect on the spread. While the intervention successfully improves home's net foreign asset position, *private* capital inflows (i.e. from intermediaries and households) are exacerbated by the intervention. This "second round" effect is particularly large when capital is fairly mobile, that is, when Γ is small (light green line). In this case, the central bank prefers smaller interest rate spreads, which, nevertheless, require larger reserve purchases.

3.2 Smooth interest rate spreads, not smooth exchange rates

A striking feature of the response in Figure 1 is that interest rate spreads are "smooth", in that τ_t is continuous over time, with $\tau_0 = \lim_{t\to\infty} \tau_t = 0$. This is true despite jumps in the foreign interest rate r^*_t and the exchange rate. The next proposition shows that this is a robust property of optimal FX interventions.[20]

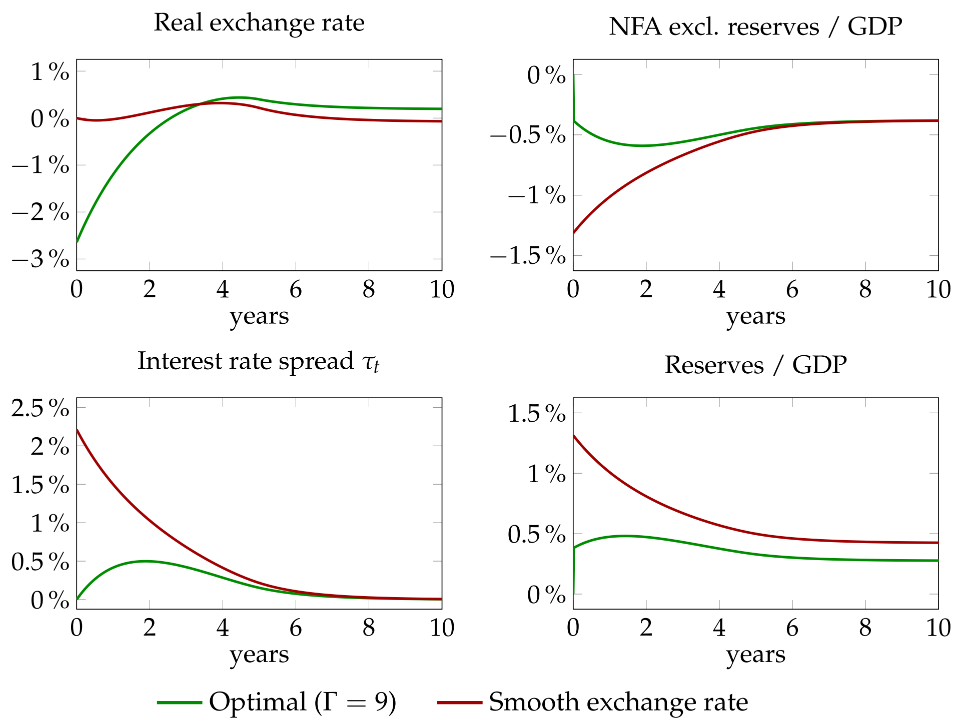
[20]This result is more general than the interest rate process considered in (21). Indeed, it holds for any integrable process for r^*_t, provided that $r_s = \rho \ \forall s > T$ for large enough T. In particular, r^*_t may jump many

Figure 1: Optimal foreign exchange interventions in response to capital inflows.



Note. This figure shows the optimal foreign exchange intervention in response to a temporary 2% cut in world interest rates, for three different degrees of financial frictions Γ in global capital flows.

Figure 2: Why smoothing exchange rates is a bad idea.



Note. This figure contrasts the optimal policy with a constrained policy that adds the requirement that the exchange rate does not jump on impact, i.e. $p_0 = \bar{p}$. The "exchange-rate-smoothing" policy is very costly since it induces large carry trade returns.

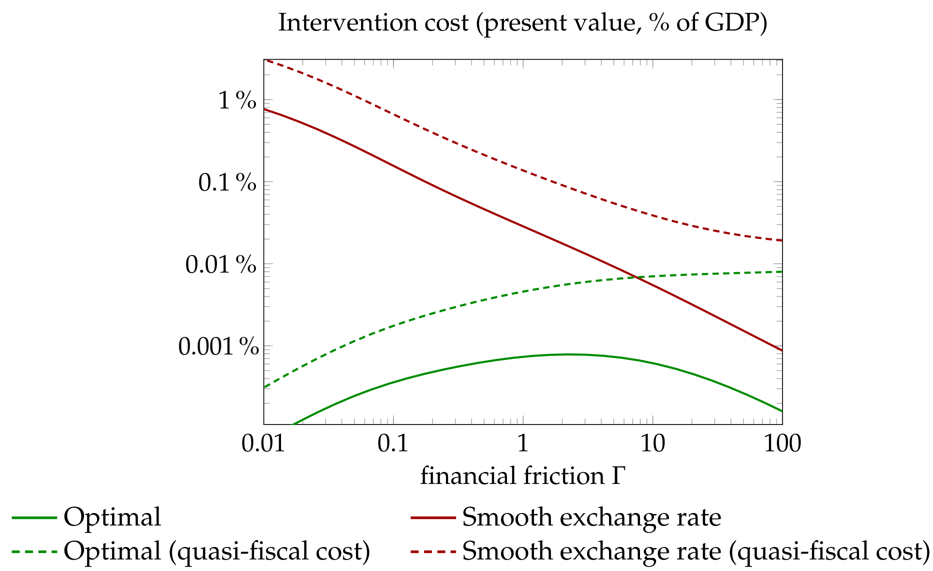
Proposition 3 (Smooth interest rate spreads.). *Under optimal FX interventions, the path of interest rate spreads τ_t is continuous in t, with $\tau_0 = \lim_{t\to\infty} \tau_t = 0$.*

This result consists of two conceptually separate parts. The first is the continuity of τ_t. This follows directly from the cost term in the countrywide budget constraint (18b): since the cost of interventions is convex in the interest rate spread τ_t, it is optimal to smooth it out over time. This bears some similarity to the logic behind the renowned tax-smoothing model of Barro (1979).

The second part is the limiting behavior of τ_t for $t = 0$ and $t \to \infty$. Initially, τ_0 is optimally equal to zero since it has a negligible effect on home households' consumption decisions while it bears a nontrivial flow cost. The limit $\lim_{t\to\infty} \tau_t$ is also optimally equal to zero as interventions far after the shock has passed do not affect household behavior during the time of the shock.

times and τ would still be continuous, start at zero and converge back to zero. This is immediately apparent from the proof of this result in Appendix C.

Figure 3: The costs from optimal FX interventions and smooth exchange rates.



Note. This figure shows the present-value budgetary cost of FX interventions in response to a capital inflow shock (see text). Green lines correspond to the fully optimal policy; red lines add the constraint that the exchange does not jump on impact, i.e. $p_0 = \bar{p}$. Solid lines represent the model-consistent cost, while dashed lines plot the "quasi-fiscal cost" typically computed in practice, which is significantly larger.

One might have expected that the central bank optimally intervenes by smoothing out the exchange rate adjustment over time, given the prevalence of such policies in practice (see e.g. Calvo and Reinhart, 2002). This is not the case. To illustrate why, amend the planning problem (19) with the additional requirement that the exchange rate be continuous at $t = 0$, that is, p_0 be equal to the pre-shock steady state price of nontradables, and consider the same inflow shock as before.

Figure 2 compares the responses with (red) and without (green) the additional smooth exchange rate requirement. As will become clear in Section 3.3, the further an intervention is in the future, the smaller the effect is on the current exchange rate. Thus, to achieve a given exchange rate target on impact, the planner has no choice but resort to substantial interventions to defend the current value of the exchange rate, generating sizable interest rates spreads. By contrast, the optimal intervention policy allows the exchange rate to jump on impact but mitigates the size of the jump by promising a stream of interventions. This is because the pecuniary externality that the planner is trying to address is inherently intertemporal in nature: the level of the exchange rate does not matter per se but rather relative to its value in the future.

Figure 2 suggests that slowing the adjustment of the exchange rate over time can be very costly, due to the large size of the associated carry trade losses. Figure 3 computes the present value of these costs, $\mathcal{C} \equiv \int_0^\infty e^{-\int_0^t r_s^* ds} \frac{1}{\Gamma} \tau_t^2 dt$, for various degrees of capital market frictions Γ. The costs associated with optimal FX interventions (green, solid) are dramatically below the costs associated with a smooth-exchange-rate policy (red, solid). Indeed, the costs associated with an optimal intervention never exceed 0.001% of GDP, whereas the costs from ensuring a smooth exchange rate can be large, on the order of 1% of GDP for an almost frictionless country ($\Gamma = 0.01$).

Two more observations are noteworthy in Figure 3. First, the costs from optimal interventions are hump-shaped in Γ and therefore largest for intermediate degrees of capital market frictions. This is because when Γ is large, there is little carry trading, while when Γ is small, interest rate spreads are also small. Second, the figure also shows the corresponding *quasi-fiscal cost*, defined as $\mathcal{C}^{qfc} = \int_0^\infty e^{-\int_0^t r_s^* ds} b_{Gt}^* \tau_t$ (dashed lines). This is the measure traditionally used to assess costs from FX interventions (e.g. Adler and Mano, 2018). As is evident, our measure of costs is one order of magnitude lower, suggesting that interventions are significantly less costly than previously thought. The reason for this discrepancy lies in the fact that quasi-fiscal costs do not incorporate that some home households may also be on the "winning" side of the carry trade. This is especially salient in the case of large Γ (no foreign intermediation), where households are the *only* beneficiaries of the carry trade: quasi-fiscal costs are still positive and large even though the economic costs for the country as a whole are zero.

3.3 Forward guidance

The principle of smooth interest rate spreads suggests that the optimal policy partly relies on interventions at a point in time when the shock has faded entirely and the world interest rate is back to r^*. In other words, the central bank engages in a kind of "FX forward guidance". The next proposition shows such forward guidance is indeed a robust property of the optimal policy.

Proposition 4 (FX forward guidance.)**.** *Optimal FX interventions remain active even after the shock has faded, that is, $\tau_t > 0$ and $b_{Gt}^* > 0$ for $t > T$ if $\Delta r^* < 0$ and $\tau_t < 0$ and $b_{Gt}^* < 0$ for $t > T$ if $\Delta r^* > 0$.*

Formally, this result is closely related to Proposition 2. The intuition can be gleaned from the Euler equation (18a). Promising interventions after the shock has passed is still a powerful way for the central bank to move consumption in all prior periods. Thus, this can be an especially cost-effective way to affect the path of the real exchange rate.

This result connects two seemingly disparate views on the effectiveness of interventions—the "signaling" and the "portfolio-balance" channels. In the empirical literature, the former is typically linked to a movement of the exchange rate at the time of the announcement, while the latter is associated with an effect when the intervention is effectively carried out. Proposition 4 suggests this approach to disentangle both views can be misleading. Our model, firmly in the portfolio-balance camp, implies a strong reaction of the exchange rate when the intervention is announced. By contrast, it does not necessarily imply a movement in the "expected" direction when the intervention occurs. For example, a reserve accumulation policy, relative to a no-intervention counterfactual, depreciates the exchange rate initially but eventually leads to real appreciation.

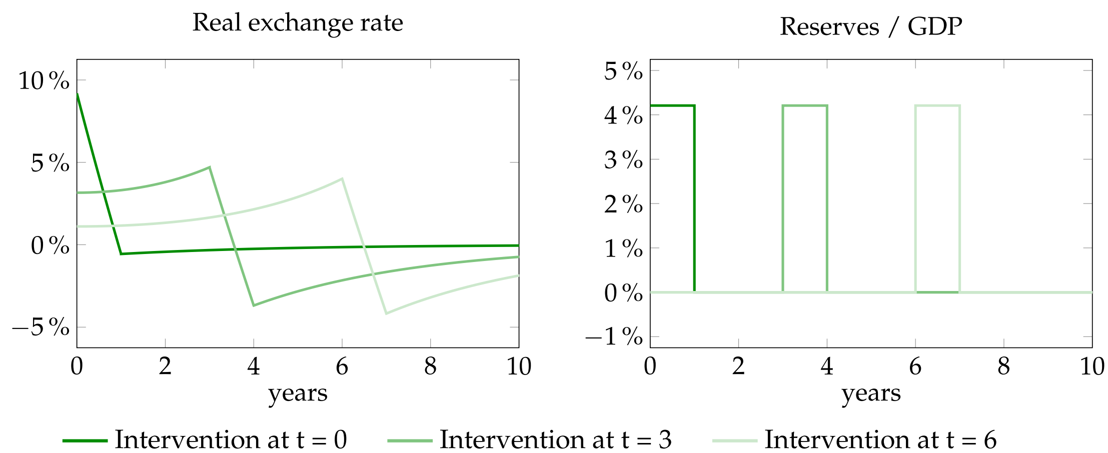
The fact that announcements of future policy actions are optimal bears a resemblance to the recent literature on forward guidance in monetary policy (Eggertsson and Woodford, 2003; Werning, 2012). One striking result in this literature is the "forward guidance puzzle"—the fact that the immediate effect of promises of future monetary policy does not decline with the horizon of those promises (Del Negro et al., 2015; McKay et al., 2016). There is no such puzzle in our model. While FX interventions in our model also operate by altering the path of real interest rates, our model does not have nominal rigidities and thus output is not demand determined. This implies that the response of home households to FX interventions is most closely comparable to the "partial equilibrium" effects of forward guidance in closed economies (Farhi and Werning, 2019). Figure 4 plots the exchange rate effect of interventions at different horizons. While there clearly is an immediate effect of the announcement, it falls rapidly with the horizon of the intervention.

One may worry that announcements can raise the attention of foreign intermediaries, lowering Γ. Interestingly, while it is correct that lower Γ makes an intervention of any given size less powerful, it also increases the elasticity of the cost to the spread. Therefore, the planner has a stronger incentive to smooth interventions over time. Indeed, Figure 1 shows that lower Γ implies a *more* (not less) back-loaded path of interest rate spreads τ_t. In other words, announcing an intervention should go hand-in-hand with greater reliance on forward guidance (see also Appendix D.1).

Of course, relying on forward guidance for FX interventions requires either sufficient credibility or commitment devices. We discuss the role of credibility and the consequences of a lack thereof next.[21]

[21] In Section 4.2, we study whether intervening in futures markets could act as a commitment tool.

Figure 4: No "puzzle" for FX forward guidance.



Note. This figure illustrates that FX forward guidance does not suffer from a "puzzle" (as in Del Negro et al. 2015; McKay et al. 2016). Future interventions are effective but less powerful than the current interventions.

3.4 Credibility is crucial

The reliance on promises about future interventions raises an obvious question: How much can and should a central bank intervene if it lacks credibility?

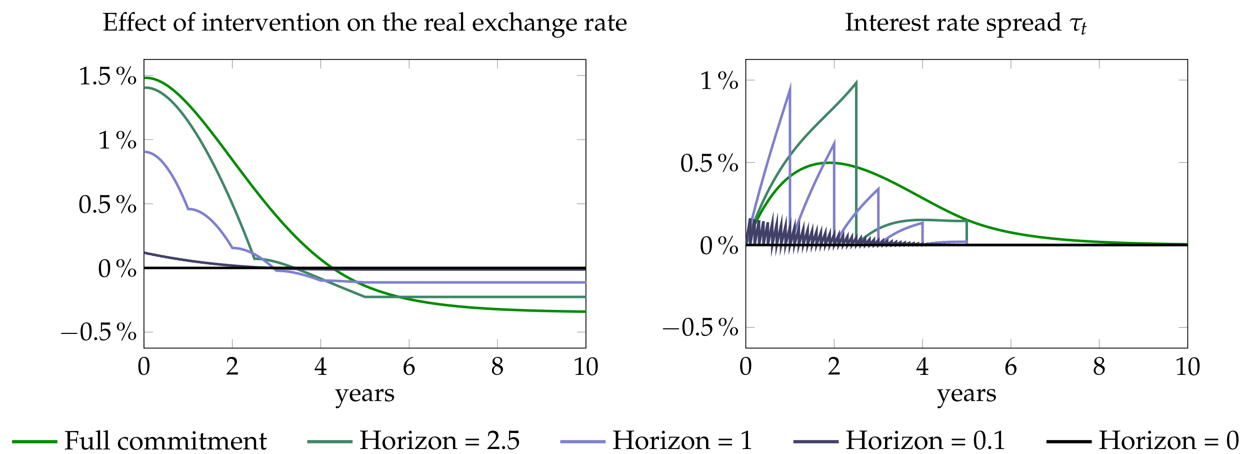
The answer to this question naturally hinges on the horizon for which a central bank can commit to its policies. For instance, one could imagine that there is a commitment horizon $\Delta > 0$ beyond which the central bank cannot commit to policies. This effectively divides time into distinct intervals $[i\Delta, (i+1)\Delta)$, $i \in \mathbb{N}$, during each of which a separate policy maker i runs the central bank. We focus here on strategies that are only allowed to depend on the asset position b inherited by i and time t. A Markov equilibrium in such a setting can then be summarized by policy functions $\tau_t = \mathcal{T}_t(b)$ and $z_t = Z_t(b)$, which jointly solve two conditions. First, $\tau_t = \mathcal{T}_t(b_{i\Delta})$ solves the planning problem

$$\mathcal{V}_{i\Delta}(b_{i\Delta}) = \max_{\tau_t, z_t} \int_{i\Delta}^{(i+1)\Delta} e^{-\rho(s-i\Delta)} V(z_s) ds + e^{-\rho\Delta} \mathcal{V}_{(i+1)\Delta}(b_{(i+1)\Delta}) \tag{22}$$

subject to the Euler equation (18a), the countrywide resource constraint (18b) and the terminal condition $z_{(i+1)\Delta} = Z_{(i+1)\Delta}(b_{(i+1)\Delta})$. Second, $z_t = Z_t(b)$ is the optimal consumption policy of Ricardian households at time t, with current asset position b, and facing interest rate spreads τ_t.

Figure 5 shows the Markov equilibrium, by simulating the response of our model, assuming various commitment horizons. As the commitment horizon shrinks, smaller and smaller interventions are optimal for the central bank. In the limit $\Delta \to 0$, the planner's

Figure 5: Smaller commitment horizons mean smaller optimal interventions.



Note. This figure illustrates that optimal FX interventions require the ability to commit. As the commitment horizon shrinks, it becomes harder to sustain large interest rate spreads τ_t resulting in smaller effects on the real exchange rate (relative to the equilibrium path). In the limit where the commitment horizon approaches zero, no intervention is optimal. (Remark: None of the paths for τ_t is ever negative, even if it may seem like that in the plot.)

optimal policy $\tau_t = \mathcal{T}_t(b)$ solves the Hamilton-Jacobi-Bellman equation

$$\rho \mathcal{V}_t(b) = \max_{\tau} V(z_t) + \mathcal{V}_t'(b) \left(r_t^* b - \frac{1}{\Gamma}\tau^2 - h(z_t) \right) \tag{23}$$

where $z_t = Z_t(b)$. This immediately yields the following proposition.

Proposition 5 (No intervention without credibility.)**.** *A central bank without credibility, $\Delta \to 0$, solving* (23)*, chooses not to intervene at all: $\tau_t = 0$ and $b_{Gt}^* = 0$ at all times t.*

This is a stark result: irrespective of the magnitude of the shock hitting the economy, the solution of the time-consistent planning problem (22) is always not to intervene at all. To gain some intuition, suppose the planner has a very small commitment horizon $\Delta > 0$ and contemplates a constant change in policy $d\tau$. This affects z_t as it changes the slope of consumption in the Euler equation. This effect is of order $\mathcal{O}(\Delta \cdot d\tau)$. Thus, the effect of the policy change on *flow* utility $V(z_t)$ is also of that order, while the effect on the *total* utility (22), integrated from t to $t + \Delta$, of the decision maker is of order $\mathcal{O}(\Delta^2 \cdot d\tau)$. By contrast, the direct effect of the policy change on flow costs is at least of order $\mathcal{O}(d\tau^2)$, with an effect on total cost of order $\mathcal{O}(\Delta \cdot d\tau^2)$, when flow costs are again integrated from t to $t + \Delta$. Thus, the benefits of such a policy change vanish relative to costs asymptotically as $\Delta \to 0$.[22]

[22]This logic can be formalized as follows. With change $d\tau$ for a small interval Δ, we have $dz_t =$

This reasoning offers an especially striking perspective on the case where $\Gamma = \infty$. A glance at the planning problem (19) reveals that when $\Gamma = \infty$, the original planning problem is already time consistent. Thus, nonzero interventions and interest rate spreads are optimal, and credibility plays no role in that case. Proposition 5 highlights that this result is "fragile": for any large but *finite* Γ, the (unique) time-consistent solution involves zero interventions and interest rate spreads.

3.5 It takes larger interventions to support an exchange rate

So far, our discussion of positive and negative shocks has been symmetric: Central banks accumulate reserves when capital flows into the economy and sell them when capital flows out. However, as we show next, there is an important *asymmetry*: for the same shock size $|\Delta r^*|$, the central bank needs to sell more reserves to stabilize the economy after capital outflows than it accumulates during capital inflows.

Proposition 6. *Let $b^*_{Gt}(\Delta r^*)$ be optimal in response to shock Δr^*. For $t > 0$, $b^*_{Gt}(\Delta r^*)$ has a kink at $\Delta r^* = 0$, with*

$$\left| \lim_{\Delta r^* \nearrow 0} \frac{db^*_{Gt}}{d\Delta r^*} \right| \leq \left| \lim_{\Delta r^* \searrow 0} \frac{db^*_{Gt}}{d\Delta r^*} \right|$$

with strict inequality if $\overline{b}^*_H > |\underline{b}^*_H|$. In words, to first order in Δr^*, it takes greater interventions to support an exchange rate (avoid depreciation) than it takes to undervalue it (avoid appreciation).

This result is a direct consequence of our assumption that $\overline{b}^*_H \geq |\underline{b}^*_H|$: when home households have an easier time saving than borrowing in foreign assets, an intervention generating a negative interest rate spread τ_t, e.g. in response to outflows, faces larger carry trades, and thus requires larger interventions. The result is especially important in emerging markets in which many households can save abroad but cannot borrow. In such countries, it can be challenging to support the exchange rate against downward pressure.[23]

$\frac{z_{t+\Delta}}{\Sigma(z_{t+\Delta})}\Delta d\tau = \mathcal{O}(\Delta d\tau)$. This follows directly from (18a) and the fact that $b_{t+\Delta}$ and $z_{t+\Delta}$ only change to the order of $\mathcal{O}(\Delta^2 d\tau)$, which is a consequence of the flow budget constraint (16). Thus, the decision maker's total utility $\mathcal{V}_t$ in (22) changes with order $\mathcal{O}(\Delta^2 d\tau)$, too. Moreover, flow costs in (16) change by at least $\frac{2}{\Gamma} d\tau_t^2 = \mathcal{O}(d\tau^2)$, which integrated from t to $t+\Delta$ gives $\mathcal{O}(\Delta d\tau^2)$.

[23] Observe that in our model, however, this does not mean that interventions generating negative spreads are more costly in welfare terms, due to their greater size. Indeed, through the lens of the model, any carry trading activity by home agents is welfare-neutral, so that the implications for welfare and spreads are symmetric around $\Delta r^* = 0$, rather than asymmetric as in Proposition 6.

3.6 Non-fundamental shocks are not costly

Policymakers often state that they intervene to curb FX volatility that is "unjustified" by fundamentals (Mohanty and Berger, 2013). For our sixth and last principle, we take this statement at face value and explore the optimal response to a "non-fundamental" shock. We model this shock as a shifter ξ_t in foreign intermediaries' demand for home bonds (i.e. a taste shock, see also Gabaix and Maggiori, 2015),

$$b_{It} = \frac{1}{\Gamma}\left(r_t + \xi_t - r^*\right).$$

Greater ξ_t implies a greater attractiveness of home bonds *only* for foreigners, i.e. it is "unjustified" from the point of view of the home country.[24] The presence of this shifter now implies that the "cost" term from nonzero interest rate spreads is

$$\tau_t b_{It} = \underbrace{\frac{1}{\Gamma}\left(\tau_t + \frac{\xi_t}{2}\right)^2}_{\text{cost term}} \underbrace{-\frac{1}{\Gamma}\frac{\xi_t^2}{4}}_{\text{monopoly benefit}}. \tag{24}$$

Here, the central bank is able to derive a *benefit* from interventions, rather than a cost. To see this, note first that the central bank can always fully avoid this shock by implementing the policy $\tau_t = 0$, which sets the cost (24) to zero and also avoids any effects on home households' behavior as the Euler equation (18a) is unchanged. But, it turns out that the central bank can do better and strictly benefit. This is because a ξ_t shock makes the central bank the monopoly supplier of bonds foreigners love. The flow benefit in equation (24) is maximized by leaning against the wind, i.e. setting $\tau_t = -\xi_t/2$. Deviations from this policy are subject to nonzero costs. Thus, the planner now seeks to smooth out $\tau_t + \xi_t/2$, rather than τ_t.

Thus, the key difference between fundamental and non-fundamental shocks is that costs are always dominated by benefits, as Proposition 7 shows.

Proposition 7. *Non-fundamental shocks ξ_t are associated with benefits rather than costs (in present value), that is,*

$$\mathcal{C}^\xi \equiv \int_0^\infty e^{-\int_0^t r_u^* du} \tau_t b_{It} \leq 0$$

Due to the optimality of the $\tau_t = 0$ allocation in the absence of a ξ_t shock, an optimal deviation from $\tau_t = 0$ must induce a first order gain in welfare.

[24] This could capture, for example, heterogeneous beliefs, a convenience yield for liquidity properties of the asset, or a change in foreigners' appetite for the risk-profile of the home bond (unmodeled).

4 Extensions and Robustness

We next present extensions and discuss the robustness of our results. First, we study the reduced form version of our planning problem in Section 2.3 and identify the crucial properties of $V(z)$ and $\Sigma(z)$ that drive our results. Appendix D uses these results to demonstrate that the principles identified in Section 3 are robust to other well known settings in international macroeconomics, e.g. terms-of-trade manipulation and aggregate demand externalities. Second, we present an extension with long-term assets and discuss whether they may be used to address the credibility issues identified in Section 3.4. Finally, we relax the assumption that the return of both bonds is denominated in dollars, and use this extension to discuss whether the interest-rate spread in the model should be interpreted as a covered- or uncovered-interest-rate-parity deviation (henceforth, CIP and UIP, respectively).

4.1 General model

In reduced form, our planning problem in Section 2.3 has a very natural representation. It involves the maximization of social welfare as a function of some notion of spending $V(z)$ subject to two constraints. First, there is an agent in the economy that is choosing the path for said spending as a function of the interest rate it perceives. This is a standard Euler equation, where $\Sigma(z)$ is the equilibrium value of the inverse elasticity of spending to the interest rate. Second, there is a standard budget constraint that includes a penalization terms for deviations between the home and the foreign interest rate.

In Appendix C we study this problem for an arbitrary increasing and concave function $V(z)$ and positive function $\Sigma(z)$. Two lessons emerge from this analysis. First, we find that a sufficient condition for leaning against the wind is

$$-V''(z)z/V'(z) \geq \Sigma(z)\,\forall z. \tag{25}$$

This condition has a very natural interpretation: it states that the planner prefers smoother expenditure paths than the agent. This condition is satisfied in all the other applications we present in Appendix D. For example, in our model with sticky prices (Appendix D.3), the exchange rate cannot appreciate after capital flows into the country and the economy booms, which can be mitigated by postponing consumption.

The second lesson that emerges from the general model in Appendix C is that the remaining principles from Section 3 carry over, regardless of whether equation (25) holds.

Thus, while the reason for intervening may differ across applications, our principles do not.

4.2 Long-term assets

Many cross-country capital flows are not in short-term assets, as assumed in our baseline model in Section 2. Here we relax this assumption. We start by deriving the correct cost term in the presence of long-term assets.

A generalized cost term. We allow agents to trade, at each time t, contracts that promise a stream of payments in the home market $\{x_{t,s}\}_s$ and in the foreign market $\{x^*_{t,s}\}_s$. Let $\pi^*_{t,s} \equiv e^{-\int_t^s r^*_u du}$ denote the state price density for an international dollar payment at time s, measured at time t. That is, if a country buys a stream of payments $\{x^*_{t,s}\}_s$ in the foreign bond market at time t, and has no other external assets or liabilities, its time-t net foreign asset position is $\text{nfa}_t = \int_t^\infty \pi^*_{t,s} x^*_{t,s} ds$. Similarly, $\pi_{t,s} \equiv e^{-\int_t^s r_u du}$ denotes the state price density for a promised payment in the home bond market at time s, measured at time t. Following the same steps as in Section 2, we arrive at a consolidated budget constraint similar to the one in (18b)

$$\int_0^\infty \pi^*_{0,t} \left\{ \frac{\alpha}{1-\alpha} p(z_t) y_N + \underline{c} - y_T + \tau_t \left(b_{It} + \int_t^\infty \pi_{t,s} b_{It,s} ds \right) \right\} = \text{nfa}_0 \tag{26}$$

where $b_{It,s}$ are the claims on home date-s payments owned by foreign intermediaries at time t and b_{It} continues to denote their position in short-term bonds.[25] Here, $b_{It} + \int_t^\infty \pi_{t,s} b_{It,s} ds$ is the net present value of the intermediary position at date t. While it appears that the present value of $\tau_t \left(b_{It} + \int_t^\infty \pi_{t,s} b_{It,s} ds \right)$ is the right cost term, this would be ignoring important revaluation effects in nfa_0. Indeed, we can split the initial net foreign asset position into what it was before the intervention was announced, nfa_{0^-}, and a revaluation term,

$$\text{nfa}_0 = \underbrace{\text{nfa}_{0^-}}_{\text{pre-intervention NFA}} + \underbrace{\int_0^\infty \left(\pi^*_{0,t} - \pi_{0,t} \right) b_{I0,t} dt}_{\text{revaluation term}} \tag{27}$$

In the case of short-term bonds, there was no revaluation term and $\text{nfa}_{0^-} = \text{nfa}_0$. With this decomposition in mind, we can rewrite (26),

$$\int_0^\infty \pi^*_{0,t} \left\{ \frac{\alpha}{1-\alpha} p(z_t) y_N + \underline{c} - y_T + \tau_t \left(b_{It} + \int_t^\infty \pi_{t,s} \Delta b_{It,s} ds \right) \right\} = \text{nfa}_{0^-} \tag{28}$$

[25] All derivations for this subsection can be found in Appendix D.2.

where $\Delta b_{It,s} \equiv b_{It,s} - b_{I0,s}$. The generalized cost term is therefore

$$\mathcal{C} = \int_0^\infty e^{-\int_0^t r_s^* ds} \tau_t \left(b_{It} + \int_t^\infty \pi_{t,s} \Delta b_{It,s} ds \right) dt$$

To interpret this expression, observe that $\int_t^\infty \pi_{t,s} \Delta b_{It,s} ds$ is the value of any *additional* long-term assets that have been purchased by intermediaries since the start of the FX interventions. Thus, while the current *stock* of short-term assets b_{It} is relevant for costs, it is only the recent *inflow* $\Delta b_{It,s} = b_{It,s} - b_{I0,s}$ of long-term assets since FX interventions began that matter.

Optimal policy. Having derived the correct general cost term, we explore the influence of long-term assets on the optimal policy next. To do this, we return to our formulation of intermediary demand from Section 2.1. Assuming position limits now apply to the total value of an intermediary's position yields a straightforward generalization of intermediary demand (7),

$$b_{It} + \int_t^\infty \pi_{t,s} b_{It,s} ds = \frac{1}{\Gamma} \left(r_t - r_t^* \right).$$

Similar to Section 4.3, the planner has the possibility of extracting a time-zero valuation gain, given by second term in (27). However, in this case the planner has commitments at different points in time, so the whole path of home interest rates matters, which makes it difficult to derive general results. One can nevertheless gain some insight into the solution by approximating the revaluation term to first order around $\tau_t = 0$,

$$\int_0^\infty \left(\pi_{0,t}^* - \pi_{0,t} \right) b_{I0,t} dt \approx \int_0^\infty e^{-\int_0^t r_u^* du} \tau_t \int_t^\infty e^{-\int_t^s r_u^* du} b_{I0,s} ds dt. \tag{29}$$

Defining $\xi_{0,t} \equiv \int_t^\infty e^{-\int_t^s r_u^* du} b_{I0,s} ds$, we see that the problem is approximately the same as the one with non-fundamental shocks (Section 3.6). Indeed, the economics are very similar. In both cases, there is a demand for the home asset paying at some time t that is not justified by fundamentals (from the point of view of the home economy). Here, the reason is that the decision to buy these assets was already made in the past, whereas in Section 3.6 the decision to buy was caused by a taste shock.

Time consistency. The date-0 revaluation effects that appear in a model with long-term assets raise an important question, namely whether those could be used to solve the time inconsistency problem that we identified in Section 3. The idea is that even though assets are linearly dependent and, hence, redundant, they are revalued differently in response to policy changes. Thus, if a planner can choose which assets are held in equilibrium, it can select a combination of assets today whose revaluation effects would "punish" future planners upon deviations (Lucas and Stokey, 1983).

Clearly, according to our previous results, if the planner were able to choose $\{b_{It,s}\}$, the time inconsistency problem could be resolved by inducing an equivalent sequence of $\{\xi_{t,s}\}$ which makes the full-commitment solution $\{\tau_t\}$ optimal for any future decision maker. In that sense, the logic in Lucas and Stokey (1983) would fully apply to our model if today's planner can influence the maturity structure of the *entire country's* liabilities.

This is not the case in our model, however, as the planner cannot directly control $b_{It,s}$. Both households and intermediaries are indifferent across all maturities, so that the composition of the revaluation term $\int_0^\infty \left(\pi_{0,t}^* - \pi_{0,t}\right) b_{I0,t} dt$ is indeterminate and cannot be influenced by the planner alone. This is the reason why the time inconsistency we identified in Section 3.4 cannot be solved by maturity management—the planner in our model cannot choose the maturity structure of the entire country.

4.3 CIP vs. UIP

So far, we have modeled the restrictions on capital mobility as constraints on cross-country holdings of otherwise identical assets, i.e. a "home" and a "foreign" bond with non-contingent dollar returns. Here, instead, we assume that the home bond b_{Ht} has returns denominated in home currency. Defining q_t to be the real exchange rate at date t,[26] we write the interest-rate spread as

$$\tau_t = r_t - \frac{\dot{q}_t}{q_t} - r_t^*. \tag{30}$$

By definition, this spread represents a UIP deviation. To study whether it purely stems from an underlying CIP deviation or not, we allow agents to trade currency forwards $\{d_{t,s}\}$. In particular, we assume that at time t agents can sign a contract that promises $f_{t,s}$ units of home currency in exchange for one dollar at time s.

We analyze two cases, depending on whether these forward markets are frictional or not.

Frictionless forward markets: Interventions create CIP deviations. Suppose forward markets are unconstrained, i.e. any agent can buy them and sell forwards without paying any participation costs or facing any position limits. Then, it must be that the expectation

[26] We adopt the convention that a higher q denotes a more depreciated home currency.

hypothesis holds, i.e. forward prices are equal to expected future spot prices

$$f_{t,s} = q_s \equiv \alpha^{\alpha}(1-\alpha)^{(1-\alpha)} p_s^{-(1-\alpha)} \tag{31}$$

or else agents would make infinite profits.

To investigate whether the spread τ_t represents an underlying CIP deviation or not, consider the following investment strategy. An intermediary sells one unit of dollar bonds at date t, buying q_t units of home currency bonds. It sells the proceeds of those bonds at $t+\Delta$ in forward markets. Thus, effectively, the intermediary purchases a home currency bond that was swapped into dollars. The return of this investment strategy at date $t+\Delta$ is the cross-currency basis,

$$\underbrace{\frac{q_t}{f_{t,t+\Delta}} e^{\int_t^{t+\Delta} r_u du} - e^{\int_t^{t+\Delta} r_u^* du}}_{\text{CIP deviation}} = \underbrace{\frac{q_t}{q_{t+\Delta}} e^{\int_t^{t+\Delta} r_u du} - e^{\int_t^{t+\Delta} r_u^* du}}_{\text{UIP deviation}} \tag{32}$$

and thus, due to frictionless forward markets (31), the UIP deviation τ_t stems entirely from a CIP deviation. Thus, τ_t should be interpreted as a CIP deviation.

Frictional forward markets: Interventions create UIP deviations. Now suppose forward markets are also constrained. That is, home agents face a constraint on the total value of their promises to deliver dollars

$$b^*_{Ht} - \int_t^{\infty} \pi^*_{t,s} d_{Ht,s} ds \in [\underline{b}^*_H, \overline{b}^*_H], \tag{33}$$

while foreign agents have to pay a participation cost to trade either home bonds b_{It} and/or currency forwards $d_{It,s}$ and face a constraint on the total value of their promises to deliver home currency,

$$b_{It} + \int_t^{\infty} \pi_{t,s} f_{t,s} d_{It,s} ds \in [-X, X]. \tag{34}$$

These restrictions capture standard constraints on open cross-currency positions faced by financial intermediaries.

As before, FX interventions induce UIP deviations given by equation (30). Are they again reflecting underlying CIP deviations? Consider again the investment strategy above. Its return is still given by the cross-currency basis (i.e. the CIP deviation) in (32). However, as $f_{t,s}$ is no longer equal to q_s, it does not necessarily equal the UIP deviation. In fact, observe that the investment strategy involves $b_{It} = q_t$ and $d_{It,t+\Delta} = -\frac{q_t}{f_{t,t+\Delta}} e^{\int_t^{t+\Delta} r_u du}$.

Substituting this into (34) shows that the investment strategy involves zero exposure to home currency and thus can be engaged in arbitrarily by intermediaries.[27] Thus, its excess return and thus the CIP deviation must be equal to zero,

$$\frac{q_t}{f_{t,t+\Delta}} e^{\int_t^{t+\Delta} r_u du} - e^{\int_t^{t+\Delta} r_u^* du} = 0.$$

Thus, in the case of frictional forward markets, the UIP deviations (30) do not reflect underlying CIP deviations.[28]

Should the spread in the model should be interpreted as a CIP or a UIP deviation in the data? Our analysis above emphasizes that to answer this question one must identify whether agents face higher restrictions shifting funds between home and foreign bond markets or across currencies. In reality, frictions on both margins exist and, depending on the situation, one may be more important than the other. For example, UIP deviations may be more important in emerging markets, which put in place several restrictions on cross-currency positions (Canales-Kriljenko, 2003), while CIP deviations may be more relevant in countries close to the zero lower bound (Amador et al., 2020).

Optimal policy. The remainder of the analysis carries over almost verbatim. Indeed, in our deterministic economy and with commitment, the only difference is that initial wealth depends on the realization of the exchange rate at $t = 0$,[29]

$$\int_0^{\infty} e^{-\int_0^t r_s^* ds} \left[\frac{\alpha}{1-\alpha} p(z_t) y_N + \underline{c} - y_T + \frac{1}{\Gamma} \tau_t^2 \right] dt = p_0^{-(1-\alpha)} \mathrm{nfa}_0.$$

If initial wealth were zero, the planning problem would be exactly the same as before. Otherwise, there is a well-known incentive to inflate away the value of debt (Fischer, 1983; Calvo, 1988). This "optimal revaluation" problem is somewhat similar to the "smooth-exchange-rate" problem studied in Section 3.2 in that the planner directly cares about the initial value of the exchange rate. For example, if the country is a debtor, the planner would trade off depreciating the currency at $t = 0$ to decrease the burden of debt with the large carry-trade costs this kind of intervention entails. As in Section 3.2, this would entail

[27] A similar argument works for home agents.

[28] Note that, if $\tau_t > 0$, the home currency forward is cheap i.e. $f_{t,t+\Delta} > q_{t+\Delta}$ and vice versa when $\tau_t < 0$. That is, when $\tau_t > 0$, there is excess demand of *both* home currency assets and home currency forwards by foreigners. On the flip side, home households would like to borrow more in dollar bonds or sell dollars forward (i.e. buy home currency forward).

[29] We assume agents are not trading forwards for this derivation. Otherwise, we would also have an additional revaluation term owing to the fact that forwards are long-term assets, as in Section 4.2.

interest rate spreads τ_t which jump at $t = 0$ and then monotonically decrease over time. Aside from the valuation effect at time zero, or whenever the planner re-optimizes, the analysis in Section 3 carries over unaltered.

The role of uncertainty. Since our model is deterministic, any UIP deviation is costly—it reflects foreign intermediaries taking the opposite position and making riskless profits. In a richer model with uncertainty, UIP deviations may also occur due to risk and not due to imperfect arbitrage. Suppose that y_{Nt} is stochastic and intermediaries value wealth in different states of the world according to some exogenous stochastic discount factor m^*. In this case, the intertemporal budget constraint becomes[30]

$$\mathbb{E}_{m^*} \int_0^\infty e^{-\int_0^t r_s^* ds} \left[\frac{\alpha}{1-\alpha} p(z_t) y_{Nt} + \underline{c} - y_T + \frac{1}{\Gamma} \tau_t^2 \right] dt = p_0^{-(1-\alpha)} \mathrm{nfa}_0$$

where $\mathbb{E}_{m^*}$ is the risk neutral measure implied by the intermediaries and the wedge is given by

$$\tau_t = r_t - \frac{1}{q_t} \frac{\mathbb{E}_{m^*} dq_t}{dt} - r_t^*.$$

In other words, only the part unjustified by risk is costly—a point also stressed by Amador et al. (2020). Thus, one should be careful when computing the costs of FX interventions from UIP deviations in the data.

More subtly, one may expect Γ to depend on policy via the exchange rate when there are financial frictions in currency markets, as in Gabaix and Maggiori (2015) and Itskhoki and Mukhin (2019). In particular, since the arbitrage is no longer riskless, foreigners may endogenously choose the size of their position X depending on exchange rate volatility. This may give rise to additional forces that are not present in our model. For example, the planner may "noise up" the exchange rate to increase Γ. In this case, while our results still illustrate one of the main forces at play, they are no longer a full description of the optimal policy, which would take into account its effect on Γ. While we think this is a very promising line of research, it is outside the scope of this paper.

[30]Note that given market incompleteness, one would also need to keep track of sequential budget constraints in the planning problem.

5 Competitive Interventions and Reserve Wars

So far, we have analyzed the optimal policy of a small open economy (SOE) against a passive rest of the world. Yet, in light of the growing popularity of FX interventions around the globe, and their potential effects on welfare, world interest rates and global capital flows, a natural next step is to study the strategic interaction between intervening economies. This is what we do in this section.

5.1 A world extension of our model

The model consists of a unit mass of (symmetric) small open economies and a unit mass of global financial intermediaries. Each of the small open economies is like the one described in Section 2. The home interest rate in a small open economy continues to be denoted by r, while r^* refers to a single world interest rate that is the rate of return of a global reserve asset (assumed to be in zero net supply). We extend our model to allow for shocks to the tradable endowment $\{y_{Tt}\}$, which will give rise to the (endogenous) innovation in the world interest rate. For simplicity, we focus on a two-period version of the model, with periods $t = 0, 1$.

Financial intermediaries. Each of the global financial intermediaries is assigned to a specific country, with respect to which it operates exactly as modeled in Section 2. It has unfettered access to the global reserve asset and limited access (subject to participation costs) to the home bonds of its assigned country. Applying the discrete-time version of the microfoundation from before, the home bond demand function comes out to be $b_I = \Gamma^{-1} \frac{r - r^*}{1 + r^*}$. Intermediaries do not have any wealth and any profits they make are assumed to be consumed by them.

Small open economies (SOE). Each SOE is inhabited by poor and Ricardian households, exactly as before, so that the planner's per-period objective can be written as $V_t(z_t)$, with z_t still being the Ricardian household's total dollar spending (net of subsistence). In each period, z_t pins down the price of nontradables $p(z_t)$, see (13), so that each SOE solves[31]

$$\max_{\{z_t\}, \tau} V_0(z_0) + \beta V_1(z_1) \tag{35}$$

[31] To simplify, we focus on the case of $\sigma = 1$ (log utility) for this section.

$$\frac{\alpha}{1-\alpha}p(z_0)y_N + \frac{1}{1+r^*}\frac{\alpha}{1-\alpha}p(z_1)y_N + \frac{1}{\Gamma}\tau^2 \leq y_{T0} - \underline{c} + \frac{1}{1+r^*}\left(y_{T1} - \underline{c}\right) \tag{36}$$

$$\frac{z_1}{z_0} = \beta(1+r^*)(1+\tau) \tag{37}$$

Here, $\frac{\alpha}{1-\alpha}p(z_t)y_N$ is the net-of-subsistence tradable spending in period t. We define the discrete-time interest rate spread as $\tau \equiv \frac{1+r}{1+r^*} - 1$. As is easily seen, the cost term from foreign intermediation is still given by $b_I\tau = \Gamma^{-1}\tau^2$ in this model. Problem (35) is therefore the exact two-period analogue of (19).

Equilibrium. We characterize symmetric equilibria conditional on the central bank policies, which we identify with the implied interest rate spread τ for convenience.

Definition 2. A *symmetric world equilibrium* with central bank FX intervention policy τ is an allocation $\{z_0, z_1\}$, where z_0, z_1 solve (35) conditional on τ; and tradable goods markets clear in both periods,

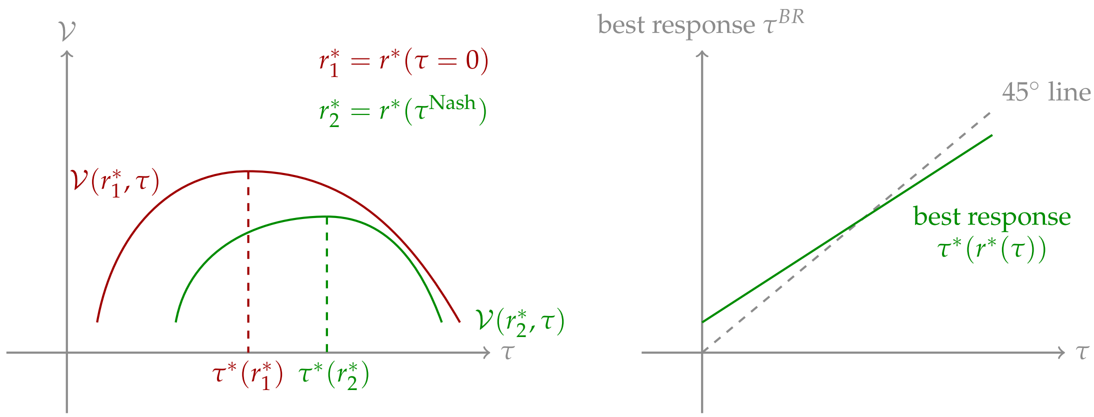
$$\frac{\alpha}{1-\alpha}p(z_0)y_N + \frac{1}{\Gamma}\tau^2 = y_{T0} - \underline{c}, \qquad \frac{\alpha}{1-\alpha}p(z_1)y_N = y_{T1} - \underline{c}. \tag{38}$$

We are especially interested in how the world equilibrium responds to a global savings glut, which we model by assuming that $y_{T1} < y_{T0}$. This assumption could for instance capture slowing future growth rates or population aging. Critical for the response of the world economy to this shock is the determination of the central banks' FX intervention policy τ. We consider two possibilities. First, τ is determined in an uncoordinated fashion ("reserve wars") as Nash equilibrium outcome. Second, τ is determined as a coordinated solution to a worldwide planning problem.

5.2 Reserve wars

When central banks do not coordinate, each takes the world interest rate r^* as given and responds by choosing the FX intervention policy τ optimally. An instructive way to study the influence of world rates on the trade-offs determining optimal interventions is to look at the country's indirect utility function $\mathcal{V}(\tau; r^*)$. We define $\mathcal{V}(\tau; r^*)$ as the objective (35) after substituting out z_0 and z_1 using constraints (36) and (37). The location of the peak of $\mathcal{V}(\tau; r^*)$ precisely corresponds to the optimal policy $\tau^*(r^*)$ in response to a given world interest rate r^*.

Figure 6: Best response to the world interest rate and the FX intervention policy of others.



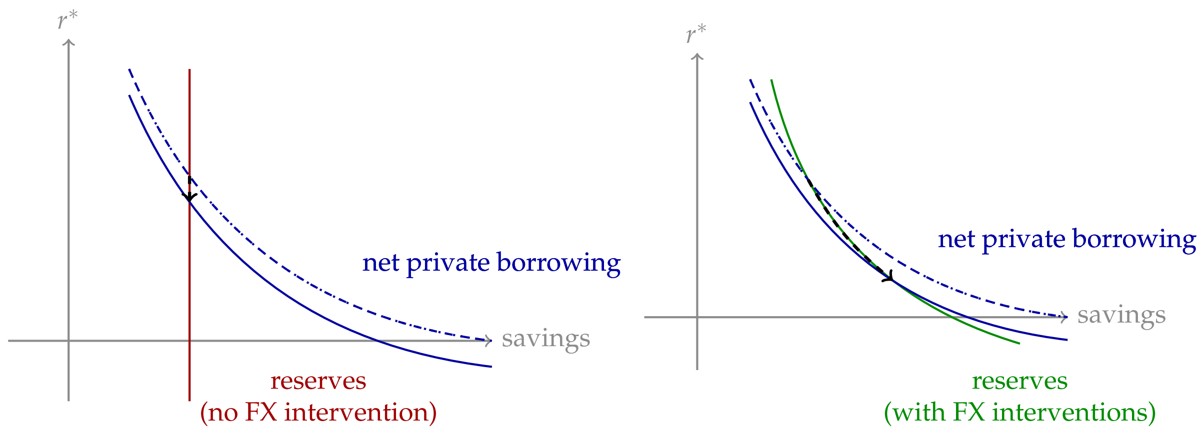
Note. The left panel illustrates that lower world interest r^* reduce an individual country's welfare and lead to larger FX interventions (higher interest rate spreads τ). The right panel illustrates that this induces a strategic complementarity between countries' FX interventions.

The left panel in Figure 6 sketches $\mathcal{V}(\tau; r^*)$ for two levels of the world interest rate r^*, one of them at the SOEs' autarky level and one of them at a lower level. The figure illustrates two ideas. First, the entire indirect utility function may in fact shift *down* in response to lower interest rates r^*. This stands in sharp contrast to a model without subsistence needs or without heterogeneity where changes in the interest rate away from a country's autarky level are always welfare improving. We prove below that this can indeed be the case if $\underline{c}$ is especially large, leading $V'(z)$ to be very elastic with respect to z. Second, the optimal response τ^* to lower world interest rates r^* is to increase interventions, that is $\tau^*(r^*)$ is decreasing in r^*. This is exactly in line with our observations in Section 3.

Obviously, the world interest rate is endogenous to the intervention policy τ of the rest of the world, so that $r^* = r^*(\tau)$. Together with the optimal policy of any individual country, this characterizes the *best response policy* $\tau^{BR}(\tau) \equiv \tau^*(r^*(\tau))$. We plot the best response policy in the right panel in Figure 6 as function of the other countries' policy τ. As can be seen, the best response is upward sloping, implying that there is a *strategic complementarity* between countries' policies. One implication of this is that any shifts in the best response schedule, e.g. when the desired savings of the countries increase, are amplified considerably by the strategic complementarity. Still, however, there is a unique intersection at the (Nash) equilibrium policy.

Another way to see the amplification is to plot world savings b_G^* by central banks against world net private borrowing $-b_H^*$, as we do in Figure 7. Clearly, they have to be equal in equilibrium. When central banks keep a fixed stock of reserves b_G^* (e.g. at

Figure 7: World savings equilibrium



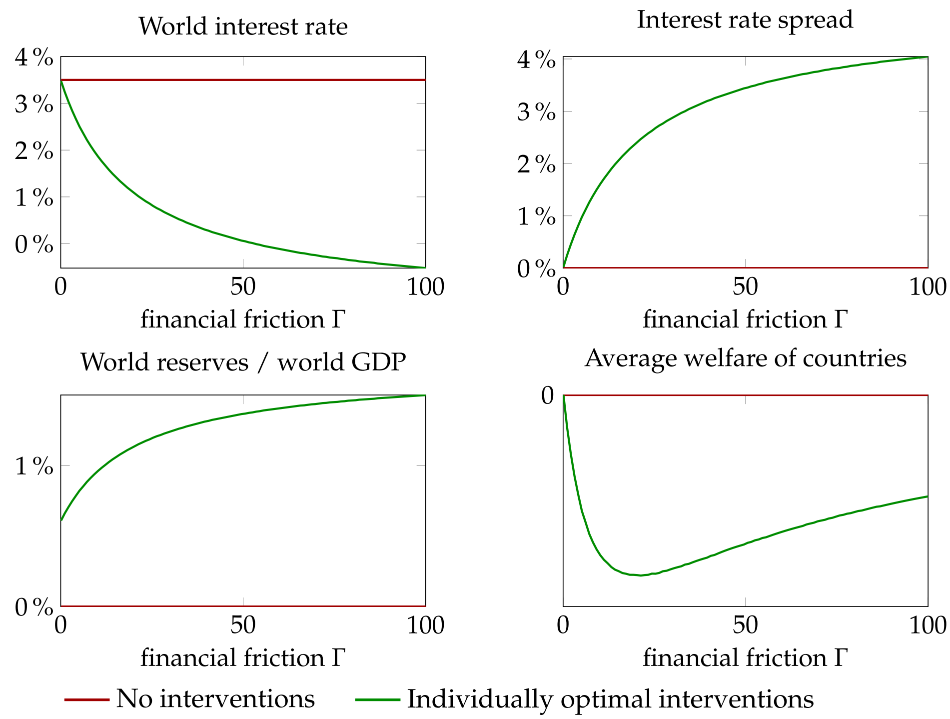
Note. This figure illustrates the world savings equilibrium. Without FX interventions, reserves are unresponsive to world interest rates r^*. With privately optimal FX interventions, countries accumulate reserves in response to lower r^*, exacerbating net private borrowing.

zero), independent of world interest rates r^*, and the world sees an increase in private saving, interest rates fall somewhat and *net* private saving remains unchanged (left panel in Figure 7). When central banks, however, implement their best response policies, increasing reserves precisely as interest rates fall, their savings supply curve becomes downward sloping and any shift in net private borrowing now has amplified effects on world interest rates and private borrowing (right panel in Figure 7).

How does this play out in practice, when the world economy experiences a global savings glut? Figure 8 simulates world interest rates r^*, interest rate spreads τ, reserves, and welfare for various levels of financial market openness, contrasting the Nash equilibrium with an economy in which all agents commit to a $\tau = 0$ policy. Clearly, both responses are equal in case of completely open financial markets, $\Gamma = 0$. Incomplete levels of openness, however, give SOEs the power to intervene in FX markets. This unleashes a "rat race" of reserve accumulation, driving world interest rates down below their no-intervention value and interest rate spreads up above zero. At the same time, no country gains from the rat race, which only benefits the intermediation sector: aggregate country welfare falls in the Nash equilibrium relative to a no-intervention world. In other words, welfare is distributed away from the intervening countries and towards the financial sector. Interestingly, the welfare losses are largest for intermediate values of financial market openness Γ. This is because interventions are costly but hardly possible when markets are relatively open and possible but hardly costly when markets are relatively closed.

We summarize these insights in the following proposition.

Figure 8: Reserve wars vs. a world without FX interventions.



Note. These figures compare an world without FX interventions (red) and the Nash equilibrium in which countries choose individually optimal FX interventions (green), as the degree of international financial frictions Γ is varied.

Proposition 8 (Reserve wars.). *Assume central banks intervene non-cooperatively. Then:*

1. *Countries buy reserves, $b_G^* > 0$, inducing a positive interest rate spread, $\tau > 0$, precisely when world interest rates are low, $r^* < \beta^{-1} - 1$. In this sense, FX interventions are strategic complements across countries.*

2. *If the elasticity of $V'(z_t)$ exceeds 2, FX interventions have negative externalities ("beggar thy neighbor") on non-intervening countries.*

3. *There exists a unique Nash equilibrium with strictly positive interest rate spreads, i.e. $\tau > 0$.*[32] *In this equilibrium, countries accumulate reserves, i.e. $b_G^* > 0$.*

Compared to a no-intervention world ($\tau = 0$ or $\Gamma = 0$), the Nash equilibrium is characterized by:

1. *Greater public outflows (reserve accumulation), and at the same time greater private inflows.*

2. *Lower world interest rates r^*.*

3. *Lower welfare of all countries.*

4. *Higher profits for the financial intermediation sector.*

5.3 Coordination

The self-defeating nature of interventions suggests that there may be gains from policy coordination among central banks. This is what we consider next. The world equilibrium is the outcome of a planning problem in which all central banks get together to maximize their joint objective (35), subject to the country-specific constraints (36), (37), but in addition take into account their effect on the world interest rate r^*, which is determined by the goods market clearing conditions (38). We can characterize the coordination outcome as follows.

Proposition 9 (Central bank cooperation.). *Assume central banks cooperate. Then they find it optimal not to intervene at all, implying zero reserves and zero interest rate spreads τ.*

In other words, the coordination outcome coincides with the no-intervention outcome in Figure 8. In that sense, *any* reserve accumulation in this model is in excess of the

[32] In this environment with simple quadratic costs we cannot rule out another Nash equilibrium in which an extreme, negative τ becomes self-fulfilling by destroying sufficiently many resources in the first period, leading to a high r^* and hence rationalizing τ. This equilibrium does not exist in any of our simulations in Figure 8.

cooperative outcome. The reason for this is that the coalition of central banks internalize that competitive devaluations are self-defeating, benefiting only the intermediation sector. Therefore, they set $\tau = 0$. This fact emphasizes that effective regulation of the international financial system may require tools to foster cooperation between central banks and prevent individually optimal but socially harmful interventions.

Symmetric vs asymmetric shocks. The need for regulation in this setting stems from the fact that, here, all countries are shocked symmetrically. If instead shocks are asymmetric (i.i.d.) across countries, there would not be a need for regulation. In fact, regulation would be harmful in that case for it would inhibit countries' abilities to use FX interventions to address distributional concerns but would not affect the world economy.

This highlights that the *nature* of the shock is crucial for efficient global regulation of FX interventions: during episodes of symmetric shocks restrictions on interventions are warranted, while during episodes of asymmetric shocks, those same restrictions can be detrimental to welfare.

6 Conclusion

FX interventions are a well-established policy instrument for both emerging and advanced economies. In this paper, we have developed a theory of such interventions. Our theory builds on the idea that choosing interventions is equivalent to managing interest rate spreads between home and foreign bonds. We emphasized that interventions are costly as they induce nonzero interest rate spreads, opening up profit opportunities for foreign carry traders. We showed these costs are convex in spreads, as larger spreads invite further speculation. This convexity lies at the heart of our optimal policy design, which we summarized in six main insights.

Our first insight is that FX interventions should lean against the wind. We showed that this is the case for four different intervention motives: in the main body of the paper we considered a distributional motive (our baseline model); in the appendix we considered models with macroeconomic-stabilization motive, sectoral heterogeneity, and a terms-of-trade-management motive. We identified the commonality across these models—excessively volatile expenditure and exchange rate paths absent interventions—and argued that FX interventions are helpful to correct precisely this class of externalities.

Among our other insights, we found that the convexity of the cost, together with the intertemporal nature of the externality, implies that interventions should induce small and smooth spreads. Furthermore, promising future interventions—FX forward guidance—is powerful, yet not subject to a "forward guidance puzzle". This induces an inherent time inconsistency problem, giving a crucial role to credibility and a rules-based approach. We also showed that the optimal policy is better approximated by a *quantity* rule rather than a *smooth exchange rate* rule.

Finally, we proposed a multi-country version of the model to address the question of spillovers of FX interventions across countries and the need for policy coordination. We concluded that coordination is, indeed, necessary to avoid wasteful competitive interventions and reserve over-accumulation. These "reserve wars" were shown to have important amplification effects on the fall of the world interest rate, hurting all countries alike. As a result, committing to a world without interventions led to a strict Pareto improvement over the Nash equilibrium in the presence of symmetric shocks.

While our results are mainly normative, we made some important positive points along the way. We derived a micro-founded version of the cost of interventions, emphasizing the importance of the maturity and residence of the holder of financial assets, and clarified the relation between the "portfolio balance" and the "signaling" channels. We hope our results will help guide future empirical research on the topic.

Our model was purposefully stylized to derive clean results and accommodate different rationales for interventions. A natural next step would be a full-blown quantitative analysis with a richer model. On the theoretical front, extending our model to allow for infrequent adjustment of portfolios, such as the one in Bacchetta and Van Wincoop (2010), and an explicit modeling of risk with incomplete markets and cross-currency financial frictions, seem like two especially relevant and promising avenues for future research.

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Appendix (for online publication)

A Calibration details

A.1 Illustrative calibration

We discipline the non-homotheticity in the model using evidence on tradable expenditure shares across the income distribution from the 2008/09 Brazilian Household Expenditure Survey (see details in Appendix A.2 below). We assume the poor household corresponds to the bottom 20% share of the population, $\mu = 0.2$, while the rich household corresponds to the remainder. We choose χ to match the implied expenditure shares, giving $\chi = 0.078$. On average poor households spend 52% of their income on tradables, while rich households spend 36% of their income on tradables. We adjust these numbers for government spending, which we for simplicity assume is nontradable and financed by the rich. After making this adjustment, we obtain that the rich spend 25% on tradables out of their gross income. These shares directly imply $\alpha = 0.051$ and $\underline{c} = 0.19$. Our results do not depend on the initial net foreign asset position, so for simplicity, we set it to zero in the steady state. This implies $y_T = 0.23$. We normalize $y_{NT} = 1$ without loss of generality.

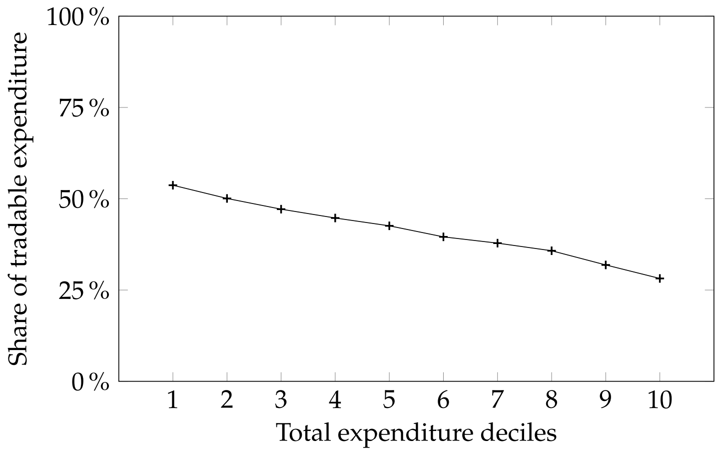
We calibrate the remaining parameters as follows. For the discount rate we pick $\rho = 0.075$, corresponding to the average 5yr treasury yield from 2000–2015 plus the average J.P. Morgan EMBI+Brazil return over the same period. We set $\sigma = 2$, a standard value. To get an idea of the relative size of home compared to foreign intermediation, we note that Brazil's private home banks operate balance sheets roughly three times the size of foreign banks' subsidiaries in Brazil (Cull et al., 2018). Interpreting balance sheet size as a rough proxy for portfolio constraints (corresponding to X in our microfoundation), this leads us to calibrate $\Gamma_H/\Gamma = 1/3$.

To calibrate the overall degree of capital immobility Γ, we rely on recent evidence from Kohlscheen and Andrade (2014), a high-frequency study of Brazilian FX interventions. They find that the announcement of a US$1 billion purchase of FX swaps leads to a depreciation of somewhere between 0.10 to 0.50%. The more conservative lower end of the spectrum therefore roughly corresponds to a 2% exchange rate movement after a 1% of GDP purchase of FX swaps (with a 1-year maturity), which is consistent with $\Gamma = 9$.[33] To explore the robustness of our predictions around this value, we also plot the model responses for values $\Gamma = 1$ and $\Gamma = 50$ below. Finally, we assume that the equilibrium without intervention, i.e. with $b^*_{Gt} = 0$ for all t, does not involve interest rate spreads, i.e. $\tau_t = 0$ for all t. This requires a sufficiently wide interval $[\underline{b}^*_H, \overline{b}^*_H]$. We choose the smallest such interval, i.e. $\overline{b}^*_H = -\underline{b}^*_H = 1.7 y_T$.[34] This assumption ensures that households have sufficient access to international asset markets that they do not need to rely on costly intermediaries.

[33] See Appendix A.3 for details on our calibration of Γ.

[34] Any sufficiently wide interval works here. The only effect a wider (but still bounded) interval has are constant offsetting shifts in the paths of reserves and home intermediaries' asset positions. In particular, the paths of interest rate spreads $\{\tau_t\}$ and the exchange rate are unaffected.

Figure 9: Tradable expenditure shares by total expenditure percentile.



A.2 Tradable expenditure shares in Brazilian household survey

Figure 9 shows tradable expenditure shares by total expenditure percentiles. The data for this plot comes from the Brazilian Household Expenditure Survey (*Pesquisa de Ornamentos Familiares*) in the years 2008/09 (sedlac). We base our classification of nontradable spending on Cravino and Levchenko (2017) and classify the following expenditure categories as tradable expenditure (the remainder is nontradable): food and non-alcoholic beverages; alcoholic beverages and narcotics; clothing and footwear; electricity, gas and other fuels; household appliances; good and services for routine household maintenance; furnishing and household equipment.

A.3 Calibration of Γ

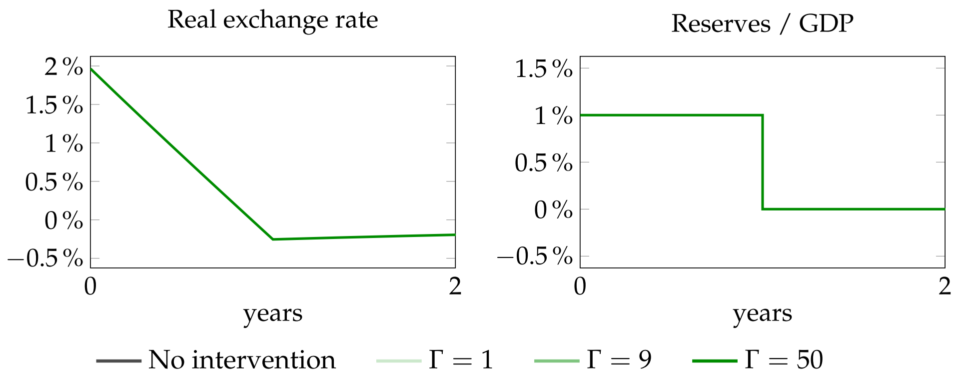
We calibrate the degree of capital immobility in our model, Γ, to match evidence from Chamon et al. (2017). To do this, we simulate an experiment where the planner increases its reserve position by 1% of GDP, for one year, for various levels of Γ, corresponding to a 1-year long reserve swap. We then determine the level of Γ for which the initial exchange rate adjustment is equal to 2%. In our case, this level is given by $\Gamma = 9$. We plot the time series of this simple experiment in Figure 10.

B Proofs for Section 2

B.1 Implementability conditions: Proposition 1

This section proves Proposition 1. It requires two directions. We start by showing that (18a) and (18b) are necessarily satisfied if $\{c^R_{Tt}, c^R_{Nt}, c^P_{Tt}, c^P_{Nt}\}$ belong to a competitive equilibrium with interest rate shocks $\{r^*_t\}$. The paragraph below Definition 1 already showed that the

Figure 10: Effects of a simple one-time reserve swap intervention.



flow version (16) of the present value budget constraint (18b) holds along a competitive equilibrium. (18a) follows directly from the Euler equation (6) and the definition of τ_t.

Now, consider the reverse direction: Given paths $\{z_t, \tau_t\}$, $\{r_t^*\}$, and an initial net foreign asset position nfa_0 that satisfy (18a) and (18b), can we always find a competitive equilibrium consisting of initial debt positions $(b_{H0}, b_{I0}, b_{IH0}, b_{G0}, b_{G0}^*)$, a central bank FX intervention policy $\{b_{Gt}, b_{Gt}^*, t_t\}$, and an allocation $\{c_{Nt}^i, c_{Tt}^i, b_{Ht}, b_{Ht}^*, b_{It}, b_{IHt}, \pi_t\}$ with prices $\{p_t, r_t\}$ such that (2)–(12) hold?

We first construct the equilibrium objects and then check optimality conditions. We can take the initial debt positions to be $b_{H0} = \text{nfa}_0$. Moreover, we define for any $T > 0$

$$b_{HT} = \int_T^\infty e^{-\int_T^t r_s^* ds} \left[\frac{\alpha}{1-\alpha} p(z_t) y_N + \underline{c} - y_T + \frac{1}{\Gamma} \tau_t^2 \right] dt, \tag{39}$$

and thus construct $b_{It} = \frac{1}{\Gamma}\tau_t$, $b_{IHt} = \frac{1}{\Gamma_H}\tau_t$, $b_{Ht}^* = \underline{b}_H^*$ if $\tau_t > 0$, $b_{Ht}^* = \overline{b}_H^*$ if $\tau_t < 0$, $b_{Ht}^* = 0$ if $\tau_t = 0$, $b_{Gt}^* + b_{Ht}^* = -b_{Gt} = b_{Ht} + b_{It} + b_{IHt}$, and $\pi_t = b_{IHt}(r_t - r_t^*)$ for each $t \geq 0$. Transfers are defined to be $t_t = r_t b_{Gt} + r_t^* b_{Gt}^*$. We let the nontradable price be defined by $p_t = p(z_t)$ using (13), define z_t^P using (3), and let consumption paths be given by (4) and (5). This concludes our construction of a candidate equilibrium. We move on to checking the equilibrium conditions.

The Euler equation (6) is equivalent to (18a). Equations (2), (3), (4), (5), (7), (8), (9), (10), and (12) hold by construction. It is straightforward to check that the nontradable market clears—that is, equation (11) holds—given our definition for the price p_t. Finally, reversing the steps in equations (14)–(16) shows that the differential (flow) version of the (39) (which is exactly (16)) implies the budget constraint (2).

B.2 The planner's objective function and concavity

In this section we derive the expression (20) for the objective function $V(z_t)$ and prove that it is strictly increasing and strictly concave, and satisfies Inada conditions. We also show $\Sigma(z)$ is positive and bounded from above.

Derivation of $V(z)$ By substituting consumption choices (4) and (5) into preferences (1), we see that per period utility of a type-i household is given by

$$V^i = u\left((\alpha z_t^i)^\alpha (p_t^{-1}(1-\alpha) z_t^i)^{1-\alpha}\right)$$

Using the fact that poor households are hand-to-mouth $z_t^P = p(z)\chi y_N/\mu + \chi y_T/\mu - \underline{c}$ and the notation that $z_t^R = z_t$, per-period utilitarian welfare is then up to a multiplicative constant

$$V(z) = (1-\mu)u\left(p(z)^{-(1-\alpha)} z\right) + \mu u\left(p(z)^{-(1-\alpha)}\left(p(z)\chi\mu^{-1} y_N + \chi\mu^{-1} y_T - \underline{c}\right)\right)$$

Factoring out $p(z)^{-(1-\alpha)(1-\sigma)}$ from this sum yields (20).

$V(z)$ **is strictly increasing** To prove this, rewrite $V(z)$ as an explicit function of z,

$$V(z) = (z - C_1)^{C_0(\sigma-1)}\left(A_0 u(z) + B_0\left(\frac{\frac{A_0}{B_0}\frac{C_1}{B_1}}{1-\frac{C_1}{B_1}}\right)^{1-\sigma} u(z - B_1)\right)$$

where $A_0 \equiv 1-\mu$, $B_0 \equiv \mu$, $C_0 \equiv 1-\alpha$, $B_1 \equiv \frac{1}{\chi(1-\mu)(1-\alpha)}(\mu\underline{c} - \chi y_T)$, and $C_1 \equiv \frac{1}{(1-\mu)}(\mu\underline{c} - \chi y_T)$. In particular, $A_0, B_0, C_0, B_1, C_1 > 0$, $C_0 < 1$, $B_1 > C_1$, and $\frac{\frac{A_0}{B_0}\frac{C_1}{B_1}}{1-\frac{C_1}{B_1}} \leq 1$ (since $\chi < \mu$). These properties alone allow us to prove monotonicity and concavity of $V(z)$. Throughout, bear in mind that the domain of z is (B_1, ∞).

After some rearranging, we can write the first derivative of V as

$$\begin{aligned}\frac{V'(z)}{(z-C_1)^{C_0(\sigma-1)-1} z^{-\sigma}} &= A_0\left((1-C_0)z - C_1\right) \\ &\quad + B_0 \underbrace{\left(\frac{\frac{A_0}{B_0}\frac{C_1}{B_1}}{1-\frac{C_1}{B_1}}\right)^{1-\sigma}\left(\frac{z-B_1}{z}\right)^{-\sigma}}_{\text{increasing in } \sigma}\left((1-C_0)z - C_1 + C_0 B_1\right) \qquad (40)\end{aligned}$$

To show that $V' > 0$, it is without loss to assume $\sigma = 0$, since the under-braced term is increasing in σ and the second term is necessarily positive,

$$(1-C_0)z - C_1 + C_0 B_1 > B_1 - C_1 > 0.$$

With $\sigma = 0$, $V'(z) > 0$ is equivalent to (after some algebra)

$$(1-C_0)B_1 > 0$$

which holds in light of $C_0 \in (0,1)$ and $B_1 > 0$. This completes the proof that V is strictly increasing.

Inada conditions for $V'(z)$ Straight from (40) we see that $\lim_{z\to B_1} V'(z) = \infty$. Moreover, note that for large z

$$V'(z) \sim z^{C_0(\sigma-1)-1} z^{-\sigma+1} = z^{-(1-C_0)\sigma - C_0}$$

where the exponent is always strictly negative since $\sigma \geq 0$ and $C_0 \in (0,1)$. This proves that $\lim_{z\to\infty} V'(z) = 0$.

$V(z)$ **is strictly concave** The second derivative of V can be rewritten as

$$\frac{V''}{z^{-\sigma-1}\left(z-C_1\right)^{C_0(\sigma-1)-2}} = A_0\mathbf{A} + B_0\left(\frac{\frac{A_0}{B_0}\frac{C_1}{B_1}}{1-\frac{C_1}{B_1}}\right)^{1-\sigma}\left(\frac{z-B_1}{z}\right)^{-\sigma-1}\mathbf{B} \tag{41}$$

where

$$\mathbf{A} = -zC_0\left((1-C_0)z - 2C_1\right) - \sigma\left(C_1 - z(1-C_0)\right)^2$$

and

$$\mathbf{B} = -(z-B_1)C_0\left((1-C_0)\left(z-B_1\right) + 2\left(B_1 - C_1\right)\right) - \sigma\left((1-C_0)z + C_0B_1 - C_1\right)^2.$$

Notice that $\mathbf{B} < 0$ for any $\sigma \geq 0$ and that $\mathbf{A}, \mathbf{B}$ both decrease in σ. As $\left(\frac{\frac{A_0}{B_0}\frac{C_1}{B_1}}{1-\frac{C_1}{B_1}}\right)^{1-\sigma}\left(\frac{z-B_1}{z}\right)^{-\sigma-1}$ is increasing in σ, it suffices to show that (41) holds for $\sigma = 0$, that is,

$$A_0\mathbf{A}(\sigma=0) + B_0\left(\frac{\frac{A_0}{B_0}\frac{C_1}{B_1}}{1-\frac{C_1}{B_1}}\right)\left(\frac{z-B_1}{z}\right)^{-1}\mathbf{B}(\sigma=0) < 0.$$

Since $z, A_0, C_0 \geq 0$, this simplifies to

$$-((1-C_0)z - 2C_1) - \frac{C_1}{B_1 - C_1}\left((1-C_0)(z-B_1) + 2(B_1 - C_1)\right) < 0$$

Notice this expression is decreasing in z, so it suffices to show the result when $z = B_1$, i.e.

$$-C_0(1-C_0)B_1 < 0,$$

which is true by virtue of $C_0 \in (0,1)$ and $B_1 > 0$. This completes the proof that V is strictly concave.

$\Sigma(z)$ is positive, continuous, and bounded from above Using the same notation as before, $\Sigma(z)$ can be rewritten as

$$\Sigma(z) = \sigma + (1-\alpha)(1-\sigma)\frac{z}{z - C_1}$$

$\Sigma(z)$ is thus continuous and monotone in z. Evaluating this expression at $z = B_1$ and $z \to \infty$ yields lower and upper bounds (depending on $\sigma \lesseqgtr 1$),

$$\Sigma(B_1) = \sigma + (1-\sigma)\frac{1-\alpha}{1-\chi(1-\alpha)}$$
$$\Sigma(\infty) = \sigma + (1-\sigma)(1-\alpha) > 0.$$

We assume parameters are such that $\Sigma(B_1) > 0$ in our baseline model. This is satisfied in our calibration.

C Proofs for Section 3

We begin by proving the following lemma.

Lemma 1. *Assume $\Gamma \in (0, \infty)$, $V(z)$ is an increasing and strictly concave function that satisfies Inada conditions, and $\Sigma(z)$ is a positive and continuous function bounded from above, i.e. $\exists K_0 > 0$ such that $\Sigma(z) < K_0\ \forall z$. When $\{r_t^*\}$ follows the path in (21), any solution $\{z_t, \tau_t\}$ to the planning problem with objective (19) and constraints (18a) and (18b) satisfies:*

1. $\tau_0 = \lim_{t\to\infty} \tau_t = 0$
2. *τ_t is continuous in t*
3. *τ_t is differentiable at $\Delta r^* = 0$*
4. *if $\frac{-V''(z)z}{V'(z)} > \Sigma(z)$ for all z, then $sign(\tau_t) = -sign(\Delta r^*)$ for all $t \in (0, \infty)$*
5. *if $\frac{-V''(z)z}{V'(z)} = \Sigma(z)$ for all z, $\tau_t = 0$ for all $t \in (0, \infty)$*
6. *if $\frac{-V''(z)z}{V'(z)} < \Sigma(z)$ for all z, then $sign(\tau_t) = sign(\Delta r^*)$ for all $t \in (0, \infty)$*

Proof. First notice that the inequality in (18b) can be relaxed to be $\leq$. This is because z_t can always be scaled up by a constant factor, which increases welfare while leaving (18a) intact.

First order conditions. The current value Hamiltonian of the planning problem is given by

$$H(z, \tau, \lambda, \psi, t) = e^{\int_0^t (r_s^* - \rho) ds} V(z) - \lambda \frac{\alpha(1-\mu)}{1-\chi(1-\alpha)} z - \lambda \frac{1}{\Gamma}\tau^2 + \psi \frac{z}{\Sigma(z)}(r^* + \tau - \rho).$$

This is an optimal control problem with a subsidiary condition, as in Gelfand and Fomin (1963). The state variable is z and has a free initial value z_0. z has a (continuously differentiable) costate ψ. $\lambda > 0$ is the multiplier on the resource constraint (18b), and τ is the control variable. For convenience, we define $\tilde{\psi} \equiv \psi \frac{z}{\Sigma(z)}$, which is also continuously differentiable, and $I_t \equiv e^{\int_0^t (r_s^* - \rho) ds}$.

H is concave in τ, with optimum at $\tau_t = \frac{\Gamma}{2\lambda} \tilde{\psi}_t$, and the costate equation for z is

$$\dot{\tilde{\psi}}_t = r_t^* \tilde{\psi}_t + \frac{z_t}{\Sigma(z_t)} \mathcal{F}(z_t, I_t)$$

where we defined $\mathcal{F}(z_t, I_t) \equiv \lambda \frac{\alpha(1-\mu)}{1-\chi(1-\alpha)} - I_t V'(z_t)$. Moreover, since z has a free initial value, $\psi_0 = \tilde{\psi}_0 = 0$. From this, it already follows that $\tau_0 = 0$ and τ_t is continuously differentiable in t.

We can rewrite the optimality conditions directly in terms of τ_t and z,

$$\dot{\tau}_t = r_t^* \tau_t + \frac{\Gamma}{2\lambda} \frac{z_t}{\Sigma(z_t)} \mathcal{F}(z_t, I_t) \tag{42a}$$

$$\Sigma(z_t) \frac{\dot{z}_t}{z_t} = r_t^* + \tau_t - \rho. \tag{42b}$$

Observe that after time $t = T$ when the shock has faded and $r_t^* = \rho$, this is a saddle-path stable system of stationary ODEs. That is, unless it is converging to a steady state, it leads to a violation of the budget constraint (since either $z \to \infty$ or $z \to B_1$ and both imply $\dot{\tau}/\tau$ is bounded from below by a positive number greater than ρ). Thus, the only optimal solution is the one where $z_t \to z^*$ and $\tau_t \to 0$, with z^* uniquely defined by $\mathcal{F}(z^*, I_T) = 0$ (this is possible since the image of $V'(z)$ is $(0, \infty)$, see Appendix B.2).

Sign of τ_t. Suppose $\Delta r^* > 0$ and $\frac{-V''(z)z}{V'(z)} > \Sigma(z)$ (the other cases are exactly analogous).[35] What can we say about the sign of τ_t? Define the positive mapping $X(z) > 0$ by[36]

$$\frac{X'(z)z}{X(z)} = \frac{d \log X(z)}{d \log z} = \Sigma(z).$$

$X(z)$ can be thought of as a measure of inverse marginal utility. In the familiar case where $\Sigma(z) = \sigma$, for instance, $X(z) = z^\sigma$. Since $X(z)$ is strictly increasing and differentiable in z, it admits an increasing and differentiable inverse, $Z(x)$. By the implicit function theorem,

$$\frac{Z'(x)x}{Z(x)} = \frac{1}{\Sigma(Z(x))}.$$

[35] Note that this proof applies to any kind of world interest rate shock $\{r_t^*\}$, as long as $r_t^* = \rho$ after $t = T$ and either $r_t^* \geq \rho$ or $r_t^* \leq \rho$ for all $t < T$. Furthermore, note that we have not used any properties of the foreign interest rate process to prove continuity and $\tau_t = 0$ and $\lim_{t\to\infty} \tau_t = 0$ other than $r_t^* = \rho \ \forall t > T$.

[36] This only pins down $X(z)$ up to a multiplicative constant which is irrelevant below.

Using $X(z)$, we do a variable substitution, from z_t to $x_t \equiv X(z_t)I_t^{-1}$, so that $z_t = Z(x_t I_t)$. This is a convenient substitution, since

$$\frac{\dot{x}_t}{x_t} = \Sigma(z_t)\frac{\dot{z}_t}{z_t} - \frac{\dot{I}_t}{I_t} = \tau_t$$

which is no longer explicitly time-dependent. We can also recast the first order condition for τ_t, (42a), in terms of x_t,

$$\dot{\tau}_t = r_t^* \tau_t + \frac{\Gamma}{2\lambda}\frac{Z(x_t I_t)}{\Sigma(Z(x_t I_t))}\mathcal{G}(x_t, I_t)$$

where $\mathcal{G}(x, I) \equiv \mathcal{F}(Z(x_t I_t), I_t)$. Similarly, to before, there exists a unique steady state x^* defined by $\mathcal{G}(x^*, I_T) = 0$ or equivalently, by $x^* = X(z^*)$.

This is a useful rewriting of the FOCs since now only the τ_t equation is non-stationary before $t = T$. After $t = T$, both equations are stationary so the state at $t = T$, (τ_T, x_T), has to lie on the stationary system's stable arm. To construct the stable arm correctly, notice that the $\dot{x} = 0$ locus is simply described by $\tau = 0$ and the $\dot{\tau} = 0$ locus is described by

$$\tau = -\frac{\Gamma}{r_t^* 2\lambda}\frac{Z(xI_t)}{\Sigma(Z(xI_t))}\mathcal{G}(x, I_t) \tag{43}$$

at time t. The relationship in (43) only crosses zero once, at $x = x_t^*$, where $x_t^* = x^*$ for $t \geq T$, and is negative (positive) for $x > x_t^*$ ($x < x_t^*$). Moreover, since I_t increases over time (due to $\Delta r^* > 0$), x_t^* falls over time as

$$\mathcal{G}_I(x, I) = -V'(z) - V''(z)Z'(xI)xI = V'(z)\left[\frac{\Sigma^V(Z(xI))}{\Sigma(Z(xI))} - 1\right] > 0 \tag{44}$$

where $\Sigma^V(z) \equiv \frac{-V''(z)z}{V'(z)}$. (44) is the case studied in this proof (the other cases are analogous).

Figure 11 illustrates these relationships in a phase diagram (thick black) and its stable arm (red line) in that case. The green line depicts the shape of the optimal trajectory that we are trying to pin down mathematically.

In a first step, we show that it can never be the case that $\tau_t \geq 0$ and $x_t > x_t^*$ for any $t > 0$. In Figure 11, this would be a state (τ_t, x_t) that lies to the top right of the time-t $x-$locus. In such a case, for any $s > t$, both $\dot{x}_s$ and $\dot{\tau}_s$ are positive and bounded away from zero, and hence the state (τ_t, x_t) would diverge to ∞. As before, τ_t would diverge at a rate that is bounded from below by a positive number greater than r_t^*, violating the budget constraint (18b).

Second, consider the possibility that for some $t > 0$,

$$(\tau_t, x_t) \in \{(\tau, x) \mid \tau \geq 0 \,\text{and}\, x \leq x_t^*\} \equiv \mathcal{X}_t.$$

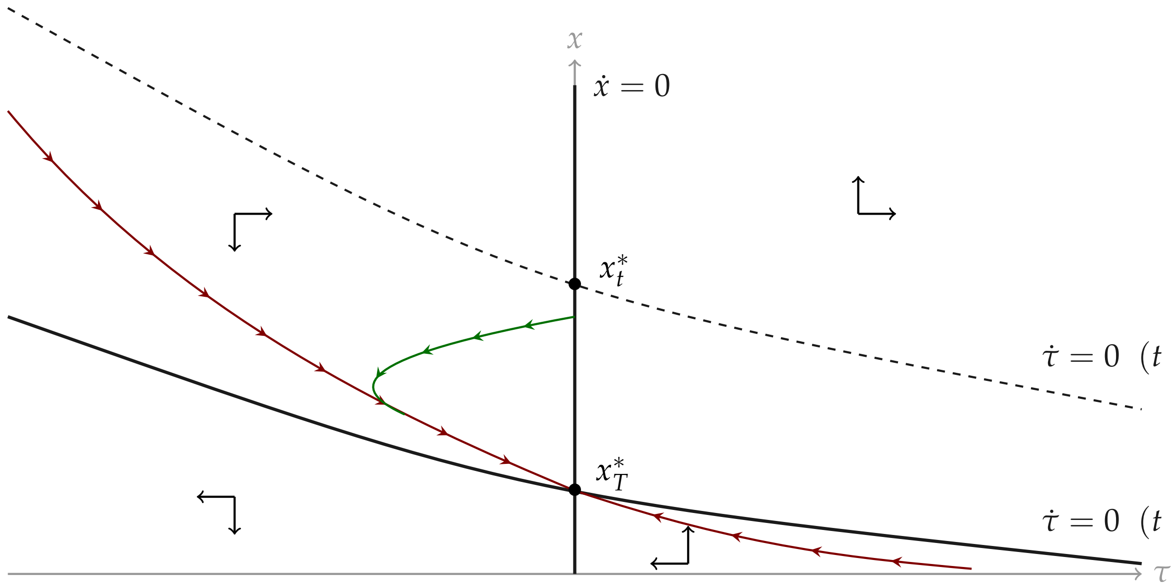


Figure 11: Describing the optimal policy in the state space for (τ, x).

Given x_t^* is decreasing in t, if $(\tau_t, x_t) \in \mathcal{X}_t$, then $(\tau_t, x_t) \in \mathcal{X}_s$ for any $s < t$ as well. In particular $(\tau_t, x_t) \in \mathcal{X}_0$. Given no path satisfying the ODEs can ever enter $\mathcal{X}_0$ (that is, $\mathcal{X}_0$, is a "source" in the vector field sense), it must hold that $(\tau_0, x_0) \in \mathrm{int}\mathcal{X}_0$ (the interior of $\mathcal{X}_0$). This contradicts the fact that $\tau_0 = 0$. Together, these two steps prove that $\tau_t \geq 0$ is impossible for any $t > 0$. Thus, $\tau_t < 0$ for $t > 0$. This concludes the proof of Lemma 1.

Differentiability with respect to Δr^*. The right hand side of the system (42) is continuously differentiable in Δr^* at $\Delta r^* = 0$. By the theorem on differentiable dependence of ODEs (see, e.g. Theorem 2.16 in Grigorian, 2007), this means that τ_t and z_t are differentiable in Δr^* for any $t \geq 0$.

Application to baseline model Appendix B.2 establishes that $V(z)$ is increasing and weakly concave, and $\Sigma(z)$ is positive, continuous and bounded from above, in the model of Section 2. Therefore, Lemma 1 applies. □

C.1 Leaning against the wind: Proposition 2

Focus on the case $\Delta r^* > 0$. Lemma 1 establishes that $\tau_t < 0$. The path for τ_t pins down the path for the net foreign asset position nfa_t since

$$\mathrm{nfa}_t = \int_t^\infty e^{-\int_t^s r_u^* du} \left[\frac{\alpha}{1-\alpha} p(z_s) y_N + \underline{c} - y_T + \frac{1}{\Gamma} \tau_s^2 \right] ds$$

as well as intermediary positions $b_{It} = \Gamma^{-1}\tau_t$ and $b_{IHt} = \Gamma_H^{-1}\tau_t$ and home household's position $b_{Ht}^* = \overline{b}_H^*$. Using the definition of the net foreign asset position and home bond market clearing (12) implies reserves must be

$$b^*_{Gt} = \text{nfa}_t + b_{It} + b_{IHt} - b^*_{Ht} \tag{45}$$

As we assumed that $\overline{b}^*_H$ is sufficiently large to ensure that an allocation with $\{\tau_t = 0\}$, which would lead to a *strictly larger* net foreign position $\text{nfa}_t^{\tau=0}$,[37] can be achieved without reserve position, that is,

$$\max_t \text{nfa}_t^{\tau=0} \leq \overline{b}^*_H$$

Thus,

$$b^*_{Gt} = \underbrace{\text{nfa}_t - \overline{b}^*_H}_{\leq 0} + \underbrace{b_{It} + b_{IHt}}_{<0} < 0$$

The predictions for the exchange rate follow directly from the fact that under the optimal policy, z_t is raised initially relative to the $\{\tau_t = 0\}$ allocation (i.e. from the fact that $\hat{z}_t$ decreases in t at the optimum in the proof of Lemma 1).

C.2 Smooth interest rate spreads: Proposition 3

This was proved in Lemma 1.

C.3 Forward guidance: Proposition 4

This was also proved in Lemma 1.

C.4 Time inconsistency: Proposition 5

This was derived in the main body of the text.

C.5 Asymmetry: Proposition 6

We proved in Lemma 1 that τ_t is differentiable in Δr^*, from which differentiability of nfa_t follows. By virtue of (45), this means:

$$\lim_{\Delta r^* \nearrow 0} \frac{\partial b^*_{Gt}}{\partial \Delta r^*} = \frac{\partial \text{nfa}_t}{\partial \Delta r^*} + \left(\Gamma^{-1} + \Gamma_H^{-1}\right) \frac{\partial \tau_t}{\partial \Delta r^*} - \underline{b}^*$$

$$\lim_{\Delta r^* \searrow 0} \frac{\partial b^*_{Gt}}{\partial \Delta r^*} = \frac{\partial \text{nfa}_t}{\partial \Delta r^*} + \left(\Gamma^{-1} + \Gamma_H^{-1}\right) \frac{\partial \tau_t}{\partial \Delta r^*} + \overline{b}^*$$

which, together with $\overline{b}^*_H > |\underline{b}^*_H|$ immediately implies the result.

[37]By design, a negative τ_t policy increases z_t initially relative to a $\{\tau_t = 0\}$ policy and therefore leads to a relatively smaller net foreign asset position.

C.6 Non-fundamental shocks: Proposition 7

Suppose the optimum had a positive present value of costs $\mathcal{C}^{\xi} = \int_0^{\infty} e^{-\int_0^t r_u^* du} \tau_t b_{It}$. The policy $\tau_t = 0$ then clearly has a lower cost, and in addition, does not distort the consumption choice by home households, implementing the first best *conditional* on a zero cost term. Therefore, the optimum cannot be one with a positive present value of costs.

D Extensions

D.1 Alternative assumptions on intermediaries' demand

Our baseline model assumed a particularly simple, linear demand schedule of foreign intermediaries. We now extend our model to allow for more general demand schedules.

Nonlinear demand. Assume the demand schedule is nonlinear and increasing, $b_{It} = g(r_t - r_t^*)$, with $g'(\tau) > 0$ and $\operatorname{sign} g(\tau) = \operatorname{sign} \tau$. In this case, the cost term in the planning problem (19) is given by

$$\tau_t b_{It} = \tau_t g(\tau_t),$$

which is no longer necessarily convex. Instead, it is globally quasi-convex and locally strictly convex around $\tau = 0$, where the second derivative is $2g'(0) > 0$. Thus, to first order, the analysis in the previous sections goes through unchanged.[38]

Local infinite elasticity. This description of intermediary demand rules out one important case, namely that where intermediary demand is locally infinitely elastic when $r_t - r_t^* = 0$. For example, this can occur in our framework if we allow a nonzero mass of foreign intermediaries to have zero participation costs (still subject to position limits). In that case, the cost term has a kink at $\tau = 0$, but remains strictly convex. The kink implies that even very small interventions have a non-negligible cost. This leads the planner to choose $\tau_t = 0$ whenever it would have chosen a τ_t close to zero without the kink, which in our experiments above means that $\tau_t \neq 0$ for t in some interval (T_1, T_2). Between T_1 and T_2, the economy behaves exactly as in the baseline model.

[38] We also conjecture that all our results generalize to the case where $g'(\tau)\tau$ is globally convex. If $\tau g(\tau)$ is sufficiently non-convex and shocks are large, the planner will start to behave as if the cost term was largely a fixed cost and therefore intervene more strongly.

D.2 Model with long-term assets

There are three important equations that change with long-term assets. First, the consolidated household budget constraint is now given by

$$\frac{\alpha}{1-\alpha}p(z_t)y_N + \underline{c} + \dot{b}_{Ht} + \dot{b}^*_{Ht} + \int_t^\infty \pi_{t,s}\partial_t b_{Ht,s}ds + \int_t^\infty \pi^*_{t,s}\partial_t b^*_{Ht,s}ds = \\ y_T + b_{Ht,t} + b^*_{Ht,t} + r_t b_{Ht} + r^*_t b^*_{Ht} + t_t + \pi_t. \tag{46}$$

Second, the central bank's budget constraint becomes

$$\dot{b_{Gt}} + \dot{b}^*_{Gt} + \int_t^\infty \pi_{t,s}\partial_t b_{Gt,s}ds + \int_t^\infty \pi^*_{t,s}\partial_t b^*_{Gt,s}ds = b_{Gt,t} + b^*_{Gt,t} + r_t b_{Gt} + r^*_t b^*_{Gt} - t_t. \tag{47}$$

Finally, bond market clearing now involves two sets of equations,

$$b_{Ht} + b_{It} + b_{IHt} + b_{Gt} = 0 \tag{48}$$

$$b_{Ht,s} + b_{It,s} + b_{IHt,s} + b_{Gt,s} = 0. \tag{49}$$

Defining the net foreign asset position,

$$\mathrm{nfa}_t \equiv b_{Ht} + b^*_{Ht} + b_{Gt} + b^*_{Gt} + \int_t^\infty \pi_{t,s}(b_{Ht,s} + b_{Gt,s})ds + \int_t^\infty \pi^*_{t,s}(b^*_{Ht,s} + b^*_{Gt,s})ds$$

observe that its derivative is just

$$\begin{aligned} \dot{\mathrm{nfa}}_t = \dot{b}_{Ht} + \dot{b}^*_{Ht} + \dot{b}_{Gt} + \dot{b}^*_{Gt} + r_t \int_t^\infty \pi_{t,s}(b_{Ht,s} + b_{Gt,s})ds \\ + r^*_t \int_t^\infty \pi^*_{t,s}(b^*_{Ht,s} + b^*_{Gt,s})ds + \int_t^\infty \pi_{t,s}\partial_t(b_{Ht,s} + b_{Gt,s})ds \\ + \int_t^\infty \pi^*_{t,s}\partial_t(b^*_{Ht,s} + b^*_{Gt,s})ds - b_{Ht,t} - b^*_{Ht,t} - b_{Gt,t} - b^*_{Gt,t} \end{aligned} \tag{50}$$

and can be substituted in when summing (46) and (47),

$$\frac{\alpha}{1-\alpha}p(z_t)y_N + \underline{c} + \dot{\mathrm{nfa}}_t = y_T + r^*_t \mathrm{nfa}_t + (r_t - r^*_t)(b_{Ht} + b_{Gt} + \int_t^\infty \pi_{t,s}(b_{Ht,s} + b_{Gt,s})ds) + \pi_t.$$

Using the market clearing conditions (48) and (49) this further simplifies to

$$\frac{\alpha}{1-\alpha}p(z_t)y_N + \underline{c} + \dot{\mathrm{nfa}}_t = y_T + r^*_t \mathrm{nfa}_t - (r_t - r^*_t)(b_{It} + b_{IHt} + \int_t^\infty \pi_{t,s}(b_{It,s} + b_{IHt,s})ds) + \pi_t,$$

and using the definition of profits, $\pi_t = (r_t - r^*_t)(b_{IHt} + \int_t^\infty \pi_{t,s} b_{IHt,s}ds)$, we arrive at

$$\frac{\alpha}{1-\alpha}p(z_t)y_N + \underline{c} + \dot{\mathrm{nfa}}_t = y_T + r^*_t \mathrm{nfa}_t - (r_t - r^*_t)(b_{It} + \int_t^\infty \pi_{t,s} b_{It,s}ds)$$

or, in present value terms,

$$\int_0^\infty \pi^*_{0,t}\left\{\frac{\alpha}{1-\alpha}p(z_t)y_N + \underline{c} - y_T + (r_t - r^*_t)(b_{It} + \int_t^\infty \pi_{t,s} b_{It,s} ds)\right\} = \text{nfa}_0.$$

Defining the net foreign asset position before the intervention as

$$\text{nfa}_{0^-} \equiv b_{H0} + b^*_{H0} + b_{G0} + b^*_{G0} + \int_0^\infty \pi^*_{0,s}(b^*_{H0,s} + b^*_{IH0,s} + b^*_{G0,s} - b_{It,s})ds$$

we arrive at (28) using the fact that $\int_0^s \pi^*_{0,t}\pi_{t,s}(r_t - r^*_t)dt = \pi^*_{0,s} - \pi_{0,s}$.

D.3 Alternative motive: Aggregate demand externalities

Our baseline model features a specific motive for intervention, namely to avoid extreme fluctuations in the consumption of poor households relative to that of Ricardian households. Here, we study a planner that chooses to defend an exchange rate peg in the presence of sticky prices.

Consider a version of the model in Section 2 with only Ricardian households ($\mu = 0$, $\chi = 0$), no subsistence needs ($\underline{c} = 0$) and log preferences ($\sigma = 1$) in which the nontradable good is produced with a linear technology $y_{Nt} = n_t$, and households experience some disutility of labor given by $v(n_t)$, i.e. utility at date t is

$$\alpha \log c_{Tt} + (1-\alpha)\log c_{Nt} - v(n_t). \tag{51}$$

Assume wages are perfectly rigid and normalized to set the home currency price of the nontradable good equal to 1, and denote the nominal exchange rate by e_t.[39] This implies that nontradable output y_{Nt} is determined by

$$e_t^{-1} y_{Nt} = (1-\alpha)z_t.$$

As is clear from (13), y_{Nt} exactly coincides with the "flexible-price output level" $y^f_N \equiv y_N$, where y_N satisfies $v'(y_N)y_N = 1-\alpha$, when $e_t = e^f_t \equiv p(z_t) = (1-\alpha)y_N^{-1}z_t$. Thus, the $\mu = \chi = \underline{c} = 0$ version of our baseline model can be nested by assuming monetary policy is implementing $e_t = e^f_t$ at all times,[40] capturing the idea that the output gap objective takes priority over the exchange rate objective in that model.

In this section, we explore the polar opposite: we assume that—for some unmodeled reason—the monetary authority has some exchange rate objective e_t. To make it stark, we assume a fixed exchange rate regime, $e_t \equiv \bar{e} = 1$, and ask: how can the planner use FX interventions to regain some monetary independence and mitigate the impact on the home

[39]We use the convention that lower values of e_t reflect a more appreciated home currency.

[40]Here, we take a shortcut and directly describe monetary policy as choosing a path for the nominal exchange rate. This can be made more formal by assuming that there is a nominal interest rate i_t such that $r_t = i_t - \dot{e}_t/e_t$. This interest rate can then be implemented using a standard interest rate rule.

economy? Examples of interventions of this sort arguably include recent interventions by Euro neighbors like Denmark, Switzerland, or the Czech Republic, which have tried to fend off appreciations and at the same time avoid being pushed into the zero, or effective, lower bound for interest rates.

In a first step, we ask which allocations can be implemented by central bank policies.[41] Fortunately, it is straightforward to show that, in fact, when stated in terms of $\{z_t, r_t\}$, the same implementability conditions as in Proposition 1 continue to hold in this economy, just with $\mu = \chi = \underline{c} = 0$. The reason is that aggregate (as well as individual) income from nontradables is still equal to $p(z_t)y_N$, even though it is now entirely caused by a larger output quantity y_{Nt} and no price response due to sticky prices.

By contrast, the objective function changes. Noting that $n_t = y_{Nt}$ and following the same steps as before, we find the planning problem to be

$$\max_{\{z_t, \tau_t\}} \int_0^\infty e^{-\rho t} \left\{ \log z_t - v((1-\alpha)z_t) \right\} dt \tag{52}$$

subject to (18a) and (18b).

Do our results carry over to this environment? Defining $V(z) \equiv \log z - v((1-\alpha)z)$, the planning problem fits into the general framework of Section 4.1. Clearly, the intertemporal elasticity of substitution implied by V is smaller than 1, which is why our results in Section 3 also hold in this model.[42]

D.4 Alternative Motive: Sectoral heterogeneity

As our next alternative model, we consider an economy that, instead of subsistence needs in consumption, features heterogeneity in sectoral employment. In particular, while a mass $1-\mu$ of households are still Ricardian, with the same endowments as before, we now assume that a fraction μ_1 of poor households (type 1) work in the tradable sector, earning χy_T, and a fraction $\mu_2 = \mu - \mu_1$ of poor households (type 2) work in the nontradable sector, earning $\chi p_t y_N$. All households share the same utility function (1), where for simplicity we focus on log preferences, $\sigma = 1$. The demands of both types of poor households are given by

$$\mu_1 c_{Tt}^{P1} = \alpha \chi y_T$$
$$\mu_2 c_{Tt}^{P2} = \alpha \chi p_t y_N$$

and similarly for nontradable goods. Thus, market clearing for nontradable goods is

$$(1-\mu)(1-\alpha)z_t + (1-\alpha)\chi y_T + (1-\alpha)\chi p_t y_N = p_t y_N$$

[41] See Amador et al. (2020) for a formal implementability result with a fixed exchange rate and a binding zero lower bound.

[42] Strictly speaking, $V(z)$ is decreasing for large enough z. It is natural to assume that we are not in this region, i.e. y_T is not that large.

which simplifies to

$$p_t = \frac{1}{y_N} \frac{1-\alpha}{1-(1-\alpha)\chi} \left((1-\mu) z_t + \chi y_T \right).$$

This is the same as (13), setting the subsistence level to zero, $\underline{c} = 0$.

The key distinction to the model in Section 2 emerges in the utility function. In this model, the planner's utility is given by, up to a constant

$$V(z) \equiv (1-\mu) \log z - (1-\alpha-\mu_1) \log p(z).$$

It is easy to see that the implementability conditions (18) are unchanged, and that $V(z)$ is increasing, concave and satisfies $-\frac{V''(z)z}{V'(z)} > 1$ when there are not too many poor households, $\mu_1 < 1-\alpha$ and $\mu_2 < \alpha$. Thus, our results in Section 3 continue to hold in this environment.

D.5 Alternative Motive: Terms-of-trade manipulation

In this section we propose a third motive for FX interventions[43] based on a dynamic terms-of-trade manipulation motive, similar to the motive in Costinot et al. (2014) and Farhi and Werning (2014). As it turns out, the planning problem of this model is again similar to the one in Section 2.3 which is why our results in Section 3 carry over unchanged.

In the simplest possible terms-of-trade model, there is a continuum of households in the home country, maximizing a common utility function $\int_0^\infty e^{-\rho t} \log(c_t) dt$, with c_t being a consumption bundle defined as $c_t = \kappa c_{Ht}^{1-\alpha} c_{Ft}^{\alpha}$. Here, c_{Ht} and c_{Ft} denote home's consumption of home and foreign goods, respectively, and $\kappa \equiv (1-\alpha)^{-(1-\alpha)} \alpha^{-\alpha} > 0$ is a positive normalization constant. We normalize the foreign good's price to 1 and refer to that numeraire as "dollars". The relative price of the home good is denoted by p_t. The per-period dollar budget constraint of the household is then given by

$$p_t c_{Ht} + c_{Ft} + \dot{b}_{Ht} + \dot{b}_{Ft} = r_t b_{Ht} + r_t^* b_{Ft} + p_t y_H + y_F + t_t + \pi_t, \tag{53}$$

where y_H is home's endowment of the home good and y_F is home's endowment of the foreign good. All other objects are as in Section 2. We denote by $q_t \equiv p_t^{-(1-\alpha)}$ the country's real exchange rate, following the convention that high values correspond to depreciated exchange rates. In this environment, home households' own a nontrivial share of the home good and exhibit home bias in their preferences. Together, these two assumptions are essential in generating the terms of trade management motive in our environment.

Maximizing utility subject to budget constraint (53) yields the following Euler equation,

$$\frac{\dot{c}_t}{c_t} = r_t - \rho + \frac{\dot{q}_t}{q_t}. \tag{54}$$

Home's total dollar expenditure is given by $q_t^{-1} c_t = p_t c_{Ht} + c_{Ft}$, which we denote, as

[43] The other two are the distributional motive of the model in Section 2 and the macroeconomic stabilization motive of the model in Section 4.1.

before, by $z_t \equiv q_t^{-1} c_t$. The optimal demand for home and foreign goods is then

$$
\begin{aligned}
p_t c_{Ht} &= (1-\alpha) z_t \\
c_{Ft} &= \alpha z_t.
\end{aligned}
\tag{55}
$$

By symmetry, foreign's demand for home goods is

$$
c_{Ht}^* = \alpha \frac{c_t^*}{p_t} \tag{56}
$$

where we assume foreign's consumption c_t^* to be equal to 1.

Replacing the household in Section 2 with the one specified here lets us set up the planning problem analogously. We obtain that the per-period objective function of the planner now takes the simple form

$$
V(z) = \log z - (1-\alpha) \log \left((1-\alpha) z + \alpha \right)
$$

and the two implementability conditions are given by

$$
\frac{\dot{z}_t}{z_t} = r_t^* + \tau_t - \rho
$$

$$
\int_0^\infty e^{-\int_0^t r_s^* ds} \left[\alpha (z_t - 1) - y_F + \frac{1}{\Gamma} \tau_t^2 \right] dt = \text{nfa}_0
$$

This is of the same form as the planning problem in Section 4.1: a strictly increasing and concave objective function, an Euler equation, and a present value budget constraint that is linear in z_t (Σ is constant and equal to one). Thus, our results in Section 3 continue to hold in this environment.

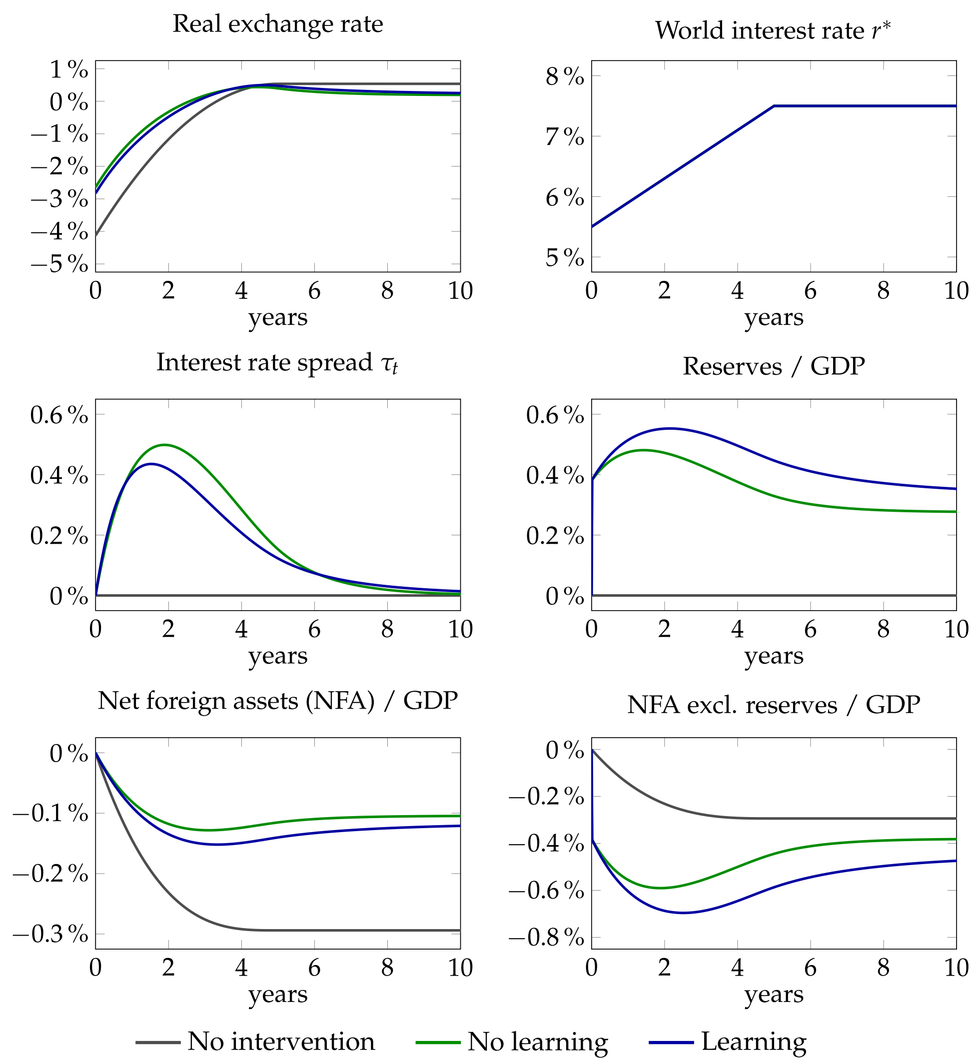
D.6 Learning

To investigate the effect of learning among intermediaries about profitable carry-trade opportunities, we repeat the shock experiment of Section 3, assuming that Γ declines over time, that is,

$$
\Gamma_t = \Gamma_0 \chi^{t/5}
$$

where we set $\chi = 0.25$. This corresponds to a fall in Γ by 75% every 5 years, or in other words, the number of active intermediaries quadruples during the period of the shock. The results of the experiment can be seen in Figure 12. The paths with and without learning are relatively close overall. They are particularly close for the real exchange rate, which, perhaps surprisingly, is achieved by a more *backloaded* path of interest rate spreads in the case *with* learning. The reason for this apparent paradox is that smoothing spreads over time becomes more desirable when Γ is low. Clearly, reserves also rise with a lower Γ. Overall, the qualitative and quantitative insights are preserved even if there is learning by

Figure 12: The effect of learning.



Note. This figure illustrates how optimal FX interventions change when intermediaries learn about the interventions and the associated carry-trade opportunities over time, implying that Γ falls over time. The black line shows an equilibrium without interventions. The green line shows optimal interventions with $\Gamma = 9$. The red line shows the optimum where we assume an exponentially decaying Γ_t, falling from by 75% every five years. Intuitively, this corresponds to a scenario where the number of active intermediaries quadruples during the period of the shock.

intermediaries.

E Proofs for Section 5

E.1 Proof of Proposition 8

Auxiliary computations. Solving the two constraints of the planning problem (35), we derive the implied expenditures given τ and r^*

$$z_0(\tau, r^*) = \frac{1}{1+\beta(1+\tau)} \frac{1}{\eta} \left(\eta_0 + \frac{\eta_1}{1+r^*} - \frac{1}{\Gamma}\tau^2 \right) \tag{57}$$

$$z_1(\tau, r^*) = \frac{\beta(1+\tau)(1+r^*)}{1+\beta(1+\tau)} \frac{1}{\eta} \left(\eta_0 + \frac{\eta_1}{1+r^*} - \frac{1}{\Gamma}\tau^2 \right). \tag{58}$$

where we defined η, η_0 such that $\frac{\alpha}{1-\alpha} p(z_t) y_N - y_{Tt} + \underline{c} = \eta z_t - \eta_t$, i.e. $\eta \equiv \frac{\alpha(1-\mu)}{1-\chi(1-\alpha)}$, $\eta_t = y_{Tt} - \underline{c} - \frac{\alpha(\chi y_{Tt} - \mu \underline{c})}{1-\chi(1-\alpha)}$. This lets us define the indirect utility function as

$$\mathcal{V}(\tau, r^*) \equiv V_0'(z_0(\tau, r^*)) + \beta V_1'(z_1(\tau, r^*)).$$

The optimal choice of τ has to satisfy the necessary first order condition

$$\mathcal{V}_\tau(\tau, r^*) = V_0'(z_0) z_{0\tau} + \beta V_1'(z_1) z_{1\tau} = 0.$$

Rearranging, we can express this as

$$\tau(1+\tau) = \frac{\beta\eta\Gamma}{2} \frac{V_1'(z_1)z_1 - V_0'(z_0)z_0(1+\tau)}{\beta V_1'(z_1)\beta(1+\tau)(1+r^*) + V_0'(z_0)} \tag{59}$$

Strategic complementarity. We prove that $\text{sign}(\tau) = \text{sign}(1-(1+r^*)\beta)$. The result for b_G^* follows from the fact that for $\tau > 0$,

$$b_G^* = \underbrace{\text{nfa} + \underline{b}_H^*}_{\geq 0} + \underbrace{b_{It}}_{>0} > 0 \tag{60}$$

as in Appendix C.1. Analogously for $\tau < 0$, $b_G^* < 0$.

In this part and the next, we focus on the case $\eta_0 = \eta_1$. We can thus drop the subindex "t" on $V_t(z)$ without risk of confusion. Consider the case $\beta(1+r^*) > 1$ and suppose $\tau \geq 0$. This immediately implies $z_1 > z_0$. Since $V'(z)z$ is decreasing in z,[44]

$$V'(z_1)z_1 < V'(z_0)z_0 \leq V'(z_0)z_0(1+\tau).$$

[44]This holds since $\frac{d\log(V'(z)z)}{d\log z} = 1 + z\frac{V''(z)}{V'(z)} < 0$ because of the assumption $\mu\underline{c} > \chi y_T$.

Equation (59) then implies $\tau < 0$, a contradiction. Hence, $\tau < 0$. Using analogous arguments one can establish that $\beta(1+r^*) < 1$ and $\beta(1+r^*) = 1$ imply $\tau > 0$ and $\tau = 0$, respectively, completing the proof.

Negative externality. To see when changes in r^* could actually hurt a country, we compute the derivative of the indirect utility $\mathcal{V}$ with respect to $1+r^*$. After some algebra, we find[45]

$$\frac{\partial \mathcal{V}}{\partial(1+r^*)} = -\frac{dV(z_0)}{d(1+r^*)}\left\{\left(\beta(1+r^*)\frac{V'(z_1)}{V'(z_0)}\right)\frac{z_1(r^*)}{z_0(r^*)} - 1\right\}. \tag{61}$$

The first interesting observation is that $\frac{\partial \mathcal{V}}{\partial(1+r^*)} = 0$ if and only if $r^* = \beta^{-1} - 1$. At that point, when r^* moves slightly, the term $B \equiv \left(\beta(1+r^*)\frac{V'(z_1)}{V'(z_0)}\right)\frac{z_1(r^*)}{z_0(r^*)}$ changes as follows (after some algebra),

$$\frac{d\log B}{d\log(1+r^*)}|_{r^*=\beta^{-1}-1} = 2 - \sigma^V$$

where $\sigma^V \equiv \frac{-V''(z)z}{V'(z)}$. This term is negative if and only if $\sigma^V > 2$, which implies $\frac{\partial \mathcal{V}}{\partial(1+r^*)}$ changes sign at $r^* = \beta^{-1} - 1$, from positive to negative. Thus, $\mathcal{V}$ has a local maximum at $r^* = \beta^{-1} - 1$, implying that changes in interest rates decrease welfare. Conversely, $\frac{\partial \mathcal{V}}{\partial(1+r^*)}$ has a local minimum if and only if $\sigma^V < 2$. This concludes this proof.

Unique Nash equilibrium. In this section we assume $\eta_0 \geq \eta_1$ (i.e. $y_{T0} \geq y_{T1}$). A Nash equilibrium requires two conditions to be satisfied. First, τ must be chosen optimality, as in (59). Second, as all countries are symmetric, none is running a current account deficit or surplus. Thus,

$$z_0(\tau, r^*) = \frac{1}{\eta}\left(\eta_0 - \frac{1}{\Gamma}\tau^2\right) \tag{62}$$

$$z_1(\tau, r^*) = \frac{1}{\eta}\eta_1 \tag{63}$$

From (62) and (63), the world interest rate is pinned down by

$$1 + r^* = \frac{1}{\beta(1+\tau)}\frac{z_1}{z_0} = \frac{1}{\beta(1+\tau)}\frac{\eta_1}{\eta_0 - \frac{1}{\Gamma}\tau^2}. \tag{64}$$

We can substitute out (64) in (59) to obtain

$$\frac{\beta V_1'(z_1)z_1 - V_0'(z_0)z_0\beta(1+\tau)}{\beta V_1'(z_1)z_1 + V_0'(z_0)z_0} z_0 = \frac{1}{\eta}\frac{2}{\Gamma}\tau(1+\tau). \tag{65}$$

[45] We omit the τ dependence as this exercise assumes $\tau = 0$ for the home country.

In this equation, τ is strictly increasing as a function of z_0,[46] with $\tau = 0$ if $z_0 = z_1$. We thus have a system of two equations, (62) and (65), and two unknowns, τ and z_0. (62) describes a decreasing relationship between z_0 and τ as long as $\tau \geq 0$ (which was assumed). Moreover, reusing the notation from Appendix B.2, an increase in y_{Tt} always reduces $V_t'(z) = \frac{A_0}{z} + \frac{B_0}{z-B_1} - \frac{C_0}{z-C_1}$ since up to a positive factor $\frac{\partial V_1'}{\partial y_T}$ is equal to

$$-B_1 B_0 \left((z - B_1) + (B_1 - c_1)\right)^2 + C_1 C_0 (z - B_1)^2$$

which is strictly negative for any $z > B_1$ because

$$B_1 B_0 > \frac{C_1}{C_0} > C_1 C_0.$$

Taken together, there exists a unique solution (τ, z_0) to the system of equations (62) and (65), and, hence, a unique Nash equilibrium.

Other results. The logic in the preceding paragraph establishes that, when $\eta_1 < \eta_0$, the interest rate spread τ is strictly positive in the Nash equilibrium. Applying (60) implies that $b_G^* > 0$ in equilibrium, but since nfa $= 0$ due to symmetry, this can only work if there are private inflows, $\underline{b}_H^* + b_{It} > 0$. Finally, all countries' welfare is lower, as is immediate from (62), the only difference between the Nash equilibrium allocation and a $\tau = 0$ allocation is that z_0 is lower, reducing welfare. Since $\tau \neq 0$, intermediaries make profits in the Nash equilibrium allocation, while they do not when $\tau = 0$.

E.2 Proof of Proposition 9

If central banks cooperate, the maximize $V_0(z_0) + \beta V_1(z_1)$ subject to (62) and (63). Per our discussion in the previous paragraph, this must imply optimality of $\tau = 0$.

[46] This follows from the implicit function theorem as the left hand side strictly increases in z_0 and decreases in τ while the right hand side strictly increases in τ.